3rd Grade

WCSD Curriculum Guides

Elementary Mathematics

Version 4: 2022/2023

Curriculum is one component of a larger mathematics instructional program in Washoe County School District (WCSD) for Kindergarten through 5th grade students. The purpose of curriculum guides are to bridge the district's K-5 Philosophy of Mathematics Education with the Nevada Academic Content Standards (NVACS) through a connection of the Curriculum Pacing Frameworks, instructional materials (*Bridges in Mathematics* or ***enVision**math**2.0***), research based instructional practices and clarification of the standards when necessary. The following describes a course of study for the specified grade for one year. ***ALL*** students must receive quality instruction in ***ALL*** grade level standards in one instructional year.

This guide is designed to be **used with the instructional materials** during planning. *This guide is not meant to supplant any portion of the instructional materials*. Teachers will continue to read through Units/Topics during instructional planning.

Guide language:

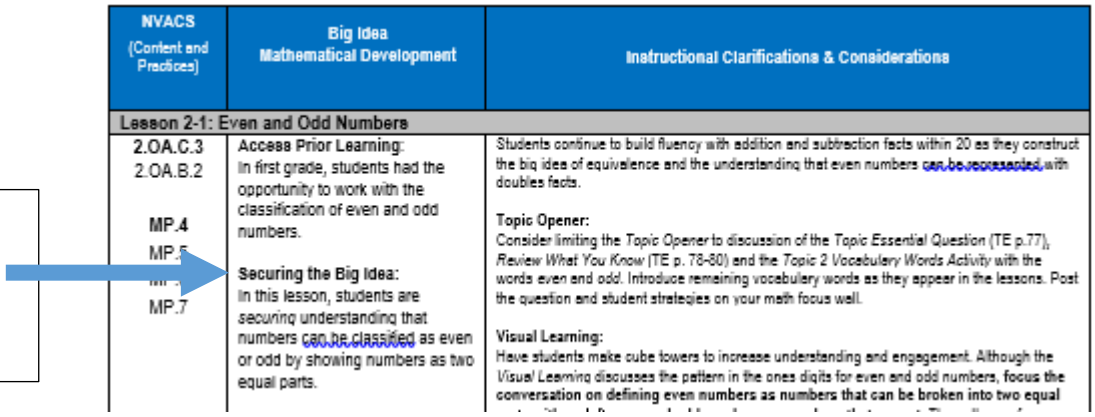
Throughout the guide the following language is used to describe the level of understanding expected at the lesson level. This language is found in the lesson-by-lesson section in the column labeled "Big Idea Mathematical Development".

Beginning: Indicates students' initial explorations with the mathematical idea(s) explored in the lesson. ***Instruction continues** to the next lesson*.

Developing: Students have worked with the mathematical ideas in previous grades or previously during the year. The focus of the lesson is to connect and build student understanding. Teachers provide intensified support to students who may exhibit misconceptions, partial understanding, no or limited understanding. ***Instruction continues** to the next lesson*.

Secure: Indicates that students have worked previously with these ideas and are expected to be at a level of secure understanding. Students with secure understanding are able to make connections and use the mathematics in a variety of situations; yet may still struggle expanding the understanding to non-routine situations. Students who are secure may still make mistakes at times; yet these students demonstrate that they have mathematical understanding with limited if any misconceptions. Students not secure in the understanding by the end of that Unit/Topic might benefit from small group intensification on these ideas. Teachers may choose to use an **F/D/E** (**F**ormative process, **D**ifferentiation or **E**nrichment) day to provide additional instructional opportunity; yet should be cautious to not spend too long exploring these ideas to ensure students have ample opportunity for instruction to ALL of the Nevada Academic Content Standards (NVACS) for mathematics.

This lesson indicates a level of secure understanding.



NVACS (Content and Practices)	Big Idea Mathematical Development	Instructional Clarifications & Considerations
Lesson 2-1: Even and Odd Numbers		
2.OA.C.3 2.OA.B.2 MP.4 MP.5 MP.7	**Access Prior Learning:** In first grade, students had the opportunity to work with the classification of even and odd numbers. **Securing the Big Idea:** In this lesson, students are securing understanding that numbers can be classified as even or odd by showing numbers as two equal parts.	Students continue to build fluency with addition and subtraction facts within 20 as they construct the big idea of equivalence and the understanding that even numbers can be represented with doubles facts. **Topic Opener:** Consider limiting the *Topic Opener* to discussion of the *Topic Essential Question* (TE p.77), *Review What You Know* (TE p. 78-80) and the *Topic 2 Vocabulary Words Activity* with the words even and odd. Introduce remaining vocabulary words as they appear in the lessons. Post the question and student strategies on your math focus wall. **Visual Learning:** Have students make cube towers to increase understanding and engagement. Although the *Visual Learning* discusses the pattern in the ones digits for even and odd numbers, focus the conversation on defining even numbers as numbers that can be broken into two equal

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2019/2020: Ben Beckam, Christin O'Keefe, Channon Toles, Denise Trakas
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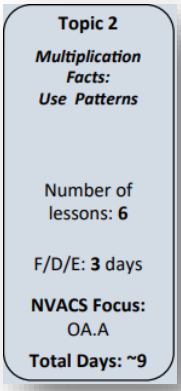
Please reference the Essential Outcomes during planning.

Note:
Please e-mail Denise Trakas (dtrakas@washoeschools.net) with any questions, concerns or potential correction suggestions.

Grade 3 Topic 2: Multiplication Facts: Use Patterns



3rd Grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Operations and Algebraic Thinking (pp. 22-28)

Prior to instruction, view the Topic 2 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 1A-1F), the Topic Planner (pp.57A-57B), all 6 lessons, and the Topic Performance Assessment (pp. 103-104A).

Mathematical Background: Read Topic 1-2 Cluster Overview/Math Background (TE, pp. 1A-1F)	**Topic Essential Question:** How can unknown multiplication facts be found using patterns and properties? *Reference Answering the Topic Essential Question (TE, pp. 101-102) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

2-1	2-3	2-4	2-2	2-5	2-6	Assessment

3 F/D/E days used strategically throughout the topic

Instructional note:

This topic focuses on *continuing* to build understanding of multiplication to meet the 2010 Nevada Academic Content Standards (NVACS) primarily focusing on 3.OA.A cluster heading, "**Represent and solve problems involving multiplication and division**" and 3.OA.D cluster heading "**Solve problems involving the four operations, and identify and explain patterns in arithmetic**." This topic also introduces the Zero and Identity properties in lesson 2-3 (3.OA.B.5).

Topic 2 focuses on understanding multiplication through an exploration of patterns. Recognizing patterns supports algebraic reasoning, relationships and leads to mathematical generalizations that help students apply understandings. Throughout Topic 2, encourage students to reason, draw conclusions, justify and generalize solutions. For example, in lesson 2-1 the *Solve & Share* provides an opportunity to explore patterns that occur when there are equal groups of 2 (there are 2 legs on *x* amount of chickens). What do students notice about the patterns? Given the conditions of the problem, can there ever be an odd number? Why or why not?

Encourage student reasoning and justifications using tools or models that demonstrate multiplication as equal groups. Spend time connecting student models to the patterns generated and connect both of these to understanding the meaning of multiplication. Building from Topic 1, ensure that the students' models represent the given equation. For example, if students build an array to demonstrate 9 x 4, the array must be represented as 9 rows of 4 stated "9 groups of 4". Or if they use a number line there should be 9 "jumps" that are in equal increments of 4. Some students may naturally start using the Commutative Property to build more efficient models or to assist their reasoning. For example, given the factors 9 and 2 students may reason about 9 groups of 2 or find it more efficient to think about 2 groups of 9. Highlight and discuss how Properties help us reason more efficiently.

Discuss and make explicit the meaning of a row and a column and how these are used to create an array. Students should have opportunities to work with concrete tools such as counters to construct the rows and columns of an array. This explicitly connects to understanding multiplication as the repeated addition of equal groups and provides a visual representation of a how a product is created. Some students may initially view multiplication as rows x columns to find the total number of counters.

Teachers should use their professional discretion to decide on the placement of Lesson 2-2. The lesson may be kept in the same order as shown in the instructional materials, or moved after lesson 2-4 as shown in this guide. See the lesson note for more information to help with this decision. Regardless of the order taught, ensure that students make the connection of using known facts (factors with 10) to derive unknown facts (factors with 9). See the instructional note in lesson 2-2 for examples.

Looking ahead to the assessment, Part A, item 2 of the Topic Assessment asks students to, "Identify any hidden question" (TE, pp. 101-102). Students in 2nd grade worked with the idea of a "hidden question" in 2-step word problems in Topics 8, 13 & 14. They will revisit this idea in Lesson 2-6. Both the Topic Assessment and the Topic Performance Assessment will provide opportunities to work at various DOK levels. Choose the assessment(s) that will provide the most information about student understanding. Consider scaffolding this resource by allowing students to work in groups throughout the topic and by ensuring opportunities for discussion, peer feedback, and revision.

Focus Math Practice 4: Model with mathematics

Focus on opportunities for students to develop Mathematical Practice 4 behaviors as this is the focus of the Math Practices and Problem Solving lesson 2-6. Reference the Teacher's Edition (TE, pp. F24 - F24A) and the Nevada Academic Content Standards for Mathematical Practice (NVACS, 2010, p.7).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
multiples Identity (One) Property of Multiplication Zero Property of Multiplication	*factor* *product* *array* *multiplication*

Additional terminology that students may need support with: patterns, relationship

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding questions: "Are students identifying patterns in multiplication? Are students using tools, strategies or models to multiply whole numbers?"

Lesson	Evidence	Look for
2-4	***Math Practices and Problem Solving*** (student work samples) Item 20	Focus CTC around the big idea: • students are identifying patterns in multiplication and explaining their thinking. • students are using various tools, strategies or models to multiply.
2-5	***Quick Check*** (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). • students understand that multiplication facts can be found by identifying patterns. • students are using various tools, strategies or models to multiply Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 101-104	Use *Scoring Guide* TE pp. 101-104A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	**Mathematical Development of the Big Idea**	**Instructional Clarifications & Considerations**
Lesson 2-1: 2 and 5 as Factors		
3.OA.A.3 **3.OA.A.1** **3.OA.D.9** MP.1 MP.2 MP.3 MP.7	**Access Prior Learning:** In Topic 1, students used skip counting, the number line, and repeated addition to think about multiplication. **Beginning of the Big Idea:** In this lesson, students are *beginning* to build their understanding and use of strategies for multiplication facts with 2 and 5 as factors with the focus on being able to skip count to find a product. Students may *begin* to develop an understanding of why when 2 is a factor the product is always even, and be able to *develop* their understanding of multiplication by connecting the multiplication equation to have an even number of groups (as in 2 x 6) or an even amount in each group (as in 6 x 2). Students may also *begin* to understand that when 5 is multiplied by an odd number, the product has a 5 in the ones	**(Possible 2-day lesson)** **Topic Opener:** Introduce the *Topic Essential Question*, "How can unknown multiplication facts be found using patterns and properties?" (TE, p. 57). Consider making this an anchor chart in your classroom. Each day new ideas are added so that students can see their ideas develop and make new connections throughout the topic. Also, consider having students complete the *Review What You Know* prior to beginning instruction on Topic 2 so that you can respond to student instructional needs using *the Item Analysis for Diagnosis and Intervention* (TE, p. 58-60). Consider introducing vocabulary as words are encountered in the lessons rather than introducing all terms at the beginning of the lesson. **Solve & Share:** Consider asking students to solve this on a blank page, white board, etc. so that the task offers multiple entry points. This will also offer you more information on how your students are understanding and seeing multiplicative situations. After student solution methods and reasoning have been shared, asking students questions such as, "What do all the products have in common?" (e.g. they're even) and "Why is that?" will help them to make generalizations and reach the understandings stated in *Beginning of the Big Idea* for when 2 is a factor. Have students confirm, clarify, or correct their ideas during the *Visual Learning Animation*. **-continues next page-**

	place. When 5 is multiplied by an even number, it has a 0 in the ones place.	**Visual Learning:** The *Visual Learning Animation* for this topic is helpful for reinforcing understanding of meanings of multiplication. If students are still struggling to understand multiplication as the joining of equal groups, view and discuss the ideas modeled as a class. Building in additional pausing points during the *Visual Learning Animation* will allow students to practice the skip counts and represent them on an open number line. **Convince Me:** Consider having a whole class discussion around the *Convince Me!* so students can continue their discovery of patterns in products when 2 and/or 5 is a factor. **Assess and Differentiate/Intervention Activity:** If time permits, teach students how to play "Quick Questions" (TE, p. 65A). All students should have the opportunity to play games that provide opportunities for practicing strategies for facts with 2 and 5 as factors. Students may also continue to play any of the games from topic 1. Consider utilizing the following question formats during practice: Example 1 Full Statement **Example Stem 1:** There are 3 rows of pictures with 2 pictures in each row. How many pictures are there? Enter your answer in the response box. Full Statement **Example Stem 2:** The pictures on a page in a picture album are in 3 rows and 2 columns. How many pictures are on the page? Enter your answer in the response box.
Lesson 2-3: Apply Properties: Multiply by 0 and 1		
3.OA.A.3 **3.OA.A.1** **3.OA.D.9** MP.1 MP.2 MP.3 MP.7	**Access Prior Learning:** In Topic 1, students developed an understanding of multiplication as joining equal groups to find the total number of objects in groups. In 2nd grade, students were secure in their understanding of the Zero Property of Addition. **Developing the Big Idea:** Students *develop* the understanding of multiplication as joining equal groups of objects to *begin* to build an understanding of the Zero Property of Multiplication and Identity (One) Property of Multiplication.	**Instructional note:** See the Instruction note at the top of this document for an explanation of ***moving lesson 2-2.*** **Solve & Share:** For concrete learners or students grappling with the misconception that multiplication always makes numbers bigger, you may consider having paper plates or bags available so that students can model having 6 groups of 0 objects. To support students' development of MP. 4 Model with mathematics, you might consider asking students how they could model or show this problem before letting them work on the *Solve & Share*. Before moving onto the *Visual Learning*, ask students "If Carlos had 6 bags with 1 apple in each bag, how many apples would he have?" Continue to question students to develop a class conjecture about multiplying by 1. Confirm, clarify, and correct this conjecture during the *Visual Learning Animation*. **Visual Learning:** Consider pausing the *Visual Learning Animation* after they introduce each property to have students test the property using counters and groups of other factors to confirm the stated property. **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 26 as this problem allows for multiple entry points and answers. When a student solves the problem, you can extend thinking by asking if that is the only answer. Students may give a generalized rule for the answer (any number greater than 4, as 4 is the minimum number needed to have a greater number of bikes than Barb's class). **Assess and Differentiate/Intervention Activity:** If time permits, you may consider replacing *Problem Solving Reading Mat* with either the games from previous topics, the game *Quick Questions* (TE, p. 65A), or the *Fluency Practice Activity* (TE, p. 97). Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p.77A).

Lesson 2-4: Multiply by 10		
3.OA.A.3 **3.OA.A.1** **3.OA.D.9** MP.2 MP.3 **MP.4** MP.7 MP.8	**Access Prior Learning:** Students have been skip counting by 10 since Kindergarten. In Topic 1 students used skip counting on the open number line to represent multiplication. **Developing the Big Idea:** Students are *developing* their understanding of patterns in multiplication by identifying a pattern when multiplying by 10.	**Solve & Share:** Consider providing students with tools, such as two-colored counters or place-value blocks to solve the problem. Students may only write the products for each week which can make identifying a pattern for multiplying by 10 difficult to see. In this event, consider asking students to write out the equations they used to solve each week and to looks for patterns in the factors and products. During the class discussion of students' solution methods and reasoning, push to get students to use place value reasoning to support their explanations of the justification offered in the *Transition to the Visual Learning Bridge* (TE, p. 79). For example, students may explain, "Since 6 x 10 means we have 6 groups of 10 miles we do not have any ones because 6 tens is 60 resulting in a 0 in the ones place for all products when 10 is a factor." Referring to 60, 70, and 80 as multiples of 10 will support students' understanding of the term multiples. **Visual Learning:** Consider pausing the video after it asks, "How many miles will Greg run to train for the race?" Discuss as a class what operation is needed to solve this question. The video will ask students this question as it's showing the 10's times tables; however, at this point connect student responses to the 10s fact table. This will support student understanding of multiplication being 'groups of'. Avoid teaching students the "zero trick" of just adding a zero to the right and instead maintain the focus on patterns that appear when multiplying by 10. Consider continuing to support students' understanding of the term "multiple(s)" by asking them to identify the multiples of 10 in the 10's times table. **Independent Practice/Math Practices and Problem Solving:** Consider including item 15 so that students have the opportunity to revisit reasoning with repeated subtraction situations. **Assess and Differentiate:** The *On-Level* and *Advanced Activity Centers* for this lesson includes 9 as a factor. Consider allowing students to play the game, but first challenge them to use what they know about multiplication with factors of 10 to develop a strategy for solving problems with 9 as a factor. Alternatively, you may wish to have students play a game from previous topics or lessons. Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p.83A). ***CTC: *Math Practices and Problem Solving*** (student work samples)
Lesson 2-2: 9 as a Factor		
3.OA.A.3 **3.OA.A.1** **3.OA.D.9** MP.1 MP.2 MP.3 MP.7	**Access Prior Learning:** In Topic 1 students came to understand multiplication as the joining together of equal groups. In *Lesson 2-4, Multiples of 10*, students identified a pattern for solving for multiplication problems with 10 as a factor. **Developing the Big Idea:** In this lesson students are *developing* their understanding of multiplication as joining equal groups and of multiples of 10 to generate a derived fact strategy for multiplication problems with 9 as a factor.	The purpose of this lesson is to explore the patterns that occur in multiplication and build mathematics curiosity and wonder at why these patterns occur. 9 as a factor reveals fascinating and unique opportunities to recognize and explore patterns beyond those that are found when looking at other factors, which may just reveal multiples or simple and easily recognizable patterns. Exploring 9 as a factor can push toward arithmetic patterns beyond those explored in earlier grades. The pattern explorations should not be seen as tricks to help students memorize the 9's facts. If using 2-2 after 2-4 then build opportunities for students to use **known** facts such as 10 x 4 (40) to **derive** a 9 fact such as 9 x 4 (I know that 10 groups of 4 is 40 so *one less group* of 4 will be 36). Moving from known to derived facts will be explored further in topic 3. **Solve & Share:** Prior to the *Solve & Share* assess student readiness by asking students to state the meaning of the equation 6 x 10 from yesterday's *Solve & Share* (e.g. 6 x 10 means we have 6 groups of 10 for a total of 60) and asking what if Duke ran 9 miles. How many will he run in 6 weeks? Consider asking a student that has direct-modeled the *Solve & Share* either with counters or pictorial representations *and* a student that used a 10 as a factor and completed a derived fact strategy to share. **-continues next page-**

Orchestrate a class discussion around these two solution methods and the reasoning used. As a class, consider how to model the math (MP.4) when using 10 as a factor. Connect the reasoning between the student's model who use a derived fact to model and that of the student that chose to direct model the 4 x 9 = 36

Look Back:
Consider discussing the *Look Back!* problem revisit ideas about the Commutative (Order) Property of Multiplication.

Visual Learning:
Since the *Visual Learning Animation* is more procedural than conceptual, consider replacing the animation by writing the 10's and 9's times tables next to each other. Facilitate a discussion to help students identify a pattern for using multiples of 10 to solve problems with 9 as a factor. Students can generalize to discover a derived fact strategy for multiplication problems with 9 as a factor. For example:

1 x 10 = 10	1 x 9 = 9
2 x 10 = 20	2 x 9 = 18
3 x 10 = 30	3 x 9 = 27
4 x 10 = 40	4 x 9 = 36
5 x 10 = 50	5 x 9 = 45
6 x 10 = 60	6 x 9 = 54
7 x 10 = 70	7 x 9 = 63
8 x 10 = 80	8 x 9 = 72
9 x 10 = 90	9 x 9 = 81
10 x 10 = 100	10 x 9 = 90

See the *Instructional Note* at the beginning of this topic for an explanation of student reasoning of this strategy.

Consider modeling the multiple by 10 and subtract the extra group using Base-10 blocks or counters and connect to the array model to support student understanding.

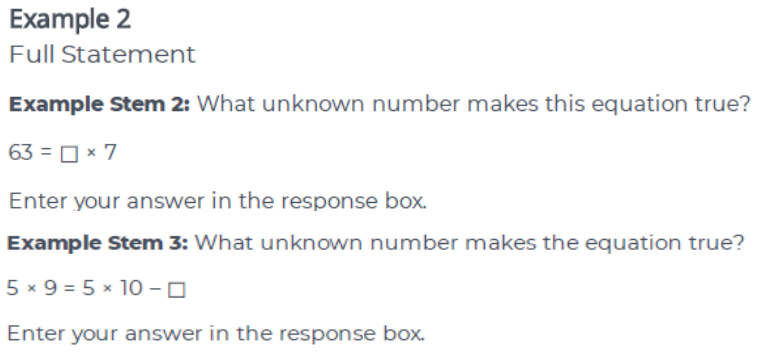
If the *Visual Learning Animation* is replaced with the above activity, question 2 in the *Guided Practice* will need to be skipped or reworded to have students describe using a multiple of 10 to solve a 9s fact.

Independent Practice/Math Practices and Problem Solving:
Notice that item 15 is the same equation students used in today's *Solve & Share*. This is an opportunity to see if students will recognize that they have worked with this problem already and it will therefore have the same product.

Assess and Differentiate/Intervention Activity:
If time permits, teach students how to play *Toss and Talk* (TE, p. 83A). All students should have the opportunity to play this game.

Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p.71A). For this activity, you can still do the *Intervention Activity Modeling 9s Facts*; however, consider replacing the worksheet with relating the models made to the finger strategy for finding multiples of 9.

Consider utilizing the following question format during practice:



Lesson 2-5: Multiplication Facts: 0, 1, 2, 5, 9, and 10		
3.OA.A.3 **3.OA.A.1** **3.OA.D.9** MP.3 **MP.4** MP.5 MP.6 MP.7	**Access Prior Learning:** In previous lessons in Topic 2 students have identified patterns that can be used as strategies for solving for facts with 0, 1, 2, 5, 9, and 10. **Developing the Big Idea:** This lesson further *develops* the understandings students began to understand in the previous lessons to apply the strategies for solving for facts with 0, 1, 2, 5, 9, and 10 as a factor.	**Solve & Share:** For the whole class discussion consider sequencing solution strategies so that the first student to share has a solution method that is similar to Stephanie's work (TE, p. 85). Consider having the second student to share be one that used solutions from having solved for the previous numbers of boxes to determine the products for the other boxes. **Visual Learning:** As students likely did not use a bar diagram to find the solution to the *Solve & Share*, the *Try It!* provides the opportunity to work with modeling the math (MP.4) using a bar diagram. It may be helpful to discuss how the bar diagram is able to represent the joining of the equal groups. **Assess and Differentiate:** Child-watch to identify students who need additional support and pull them into a small group to complete the Intervention Activity (TE, p.23A). ***CTC: *Quick Check*** (digital platform) Consider utilizing the following question formats during practice: Example 4 Full Statement **Example Stem 4:** Claire arranges 6 pictures into an array with 3 rows. How many columns of pictures are in the array? Enter your answer in the response box. Example 6 Full Statement **Example Stem 6:** Lisa arranges 6 pictures into an array with 2 columns. How many rows of pictures are in the array? Enter your answer in the response box.
Lesson 2-6: Math Practices and Problem Solving- Model with Math		
3.OA.A.3 **MP.4** MP.1 MP.2 MP.3 MP.5	**Access Prior Learning:** In this topic students have identified patterns that can be used as strategies for solving for facts with 0, 1, 2, 5, 9, and 10. Lesson 1-7 involved students solving for a 2-step word problem. Students also worked with 2-step word problems involving addition and subtraction and the idea of a "hidden problem" in 2nd grade Topics 8, 13 & 14. **Developing the Big Idea:** In this lesson, students are *developing* their understanding of multiplication as joining equal groups and the use of patterns for multiplying with 0, 1, 2, 5, 9, and 10.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 4. Refer to the Math Practices and *Problem Solving Handbook* (TE p. F24-F24A, F29) for suggestions on how to develop, connect and assess this Math Practice. Also, reference the handbook in the student edition (SE, p. F24). **Solve & Share:** Consider reintroducing MP. 4 Thinking Habits (SE, p. F24) before introducing the *Solve & Share*. Restating that an equation is an example of MP.4 Modeling the Math can be a good reminder. Many students are under the misconception that MP. 4 means they must show a drawing or concrete representation of the math. While having a drawn or concrete representation of the math can make for a stronger argument (MP.3), it is not necessary for modeling mathematical situations. Consider using the time when students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.4 that are listed in the *Math Practices and Problem Solving Handbook* (p. F24A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice. **Convince Me:** Consider assigning the *Convince Me!* as it offers another opportunity to work with MP.4 and assess for behaviors attributed to this math practice. **Assess and Differentiate:** If time permits, consider assigning the *Math and Science Activity* (TE, p. 95A) as this relates the mathematics in this topic to a real world context. Child-watch to identify students who need additional support and pull them into a small group to complete the Intervention Activity (TE, p.95A).

References

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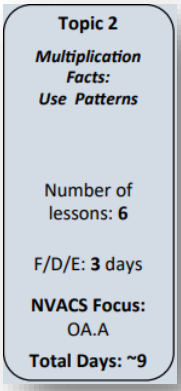
Grade 3 Topic 2: Multiplication Facts: Use Patterns



[3rd Grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: Operations and Algebraic Thinking (pp. 22-28)

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Topic 2 focuses on understanding multiplication through an exploration of patterns. Recognizing patterns supports algebraic reasoning, relationships and leads to mathematical generalizations that help students apply understandings. Throughout Topic 2, encourage students to reason, draw conclusions, justify and generalize solutions. For example, in lesson 2-1 the *Solve & Share* provides an opportunity to explore patterns that occur when there are equal groups of 2 (there are 2 legs on *x* amount of chickens). What do students notice about the patterns? Given the conditions of the problem, can there ever be an odd number? Why or why not?

Encourage student reasoning and justifications using tools or models that demonstrate multiplication as equal groups. Spend time connecting student models to the patterns generated and connect both of these to understanding the meaning of multiplication. Building from Topic 1, ensure that the students' models represent the given equation. For example, if students build an array to demonstrate 9 x 4, the array must be represented as 9 rows of 4 stated "9 groups of 4". Or if they use a number line there should be 9 "jumps" that are in equal increments of 4. Some students may naturally start using the Commutative Property to build more efficient models or to assist their reasoning. For example, given the factors 9 and 2 students may reason about 9 groups of 2 or find it more efficient to think about 2 groups of 9. Highlight and discuss how Properties help us reason more efficiently.

Discuss and make explicit the meaning of a row and a column and how these are used to create an array. Students should have opportunities to work with concrete tools such as counters to construct the rows and columns of an array. This explicitly connects to understanding multiplication as the repeated addition of equal groups and provides a visual representation of a how a product is created. Some students may initially view multiplication as rows x columns to find the total number of counters.

Teachers should use their professional discretion to decide on the placement of Lesson 2-2. The lesson may be kept in the same order as shown in the instructional materials, or moved after lesson 2-4 as shown in this guide. See the lesson note for more information to help with this decision. Regardless of the order taught, ensure that students make the connection of using known facts (factors with 10) to derive unknown facts (factors with 9). See the instructional note in lesson 2-2 for examples.

Looking ahead to the assessment, Part A, item 2 of the Topic Assessment asks students to, "Identify any hidden question" (TE, pp. 101-102). Students in 2nd grade worked with the idea of a "hidden question" in 2-step word problems in Topics 8, 13 & 14. They will revisit this idea in Lesson 2-6. Both the Topic Assessment and the Topic Performance Assessment will provide opportunities to work at various DOK levels. Choose the assessment(s) that will provide the most information about student understanding. Consider scaffolding this resource by allowing students to work in groups throughout the topic and by ensuring opportunities for discussion, peer feedback, and revision.

Focus Math Practice 4: Model with mathematics

Focus on opportunities for students to develop Mathematical Practice 4 behaviors as this is the focus of the Math Practices and Problem Solving lesson 2-6. Reference the Teacher's Edition (TE, pp. F24 - F24A) and the Nevada Academic Content Standards for Mathematical Practice (NVACS, 2010, p.7).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
multiples Identity (One) Property of Multiplication Zero Property of Multiplication	*factor* *product* *array* *multiplication*

Additional terminology that students may need support with: patterns, relationship

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding questions: "Are students identifying patterns in multiplication? Are students using tools, strategies or models to multiply whole numbers?"

Lesson	Evidence	Look for
2-4	***Math Practices and Problem Solving*** (student work samples) Item 20	Focus CTC around the big idea: • students are identifying patterns in multiplication and explaining their thinking. • students are using various tools, strategies or models to multiply.
2-5	***Quick Check*** (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). • students understand that multiplication facts can be found by identifying patterns. • students are using various tools, strategies or models to multiply Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 101-104	Use *Scoring Guide* TE pp. 101-104A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	**Mathematical Development of the Big Idea**	**Instructional Clarifications & Considerations**
Lesson 2-1: 2 and 5 as Factors		
3.OA.A.3 **3.OA.A.1** **3.OA.D.9** MP.1 MP.2 MP.3 MP.7	**Access Prior Learning:** In Topic 1, students used skip counting, the number line, and repeated addition to think about multiplication. **Beginning of the Big Idea:** In this lesson, students are *beginning* to build their understanding and use of strategies for multiplication facts with 2 and 5 as factors with the focus on being able to skip count to find a product. Students may *begin* to develop an understanding of why when 2 is a factor the product is always even, and be able to *develop* their understanding of multiplication by connecting the multiplication equation to have an even number of groups (as in 2 x 6) or an even amount in each group (as in 6 x 2). Students may also *begin* to understand that when 5 is multiplied by an odd number, the product has a 5 in the ones	**(Possible 2-day lesson)** **Topic Opener:** Introduce the *Topic Essential Question*, "How can unknown multiplication facts be found using patterns and properties?" (TE, p. 57). Consider making this an anchor chart in your classroom. Each day new ideas are added so that students can see their ideas develop and make new connections throughout the topic. Also, consider having students complete the *Review What You Know* prior to beginning instruction on Topic 2 so that you can respond to student instructional needs using *the Item Analysis for Diagnosis and Intervention* (TE, p. 58-60). Consider introducing vocabulary as words are encountered in the lessons rather than introducing all terms at the beginning of the lesson. **Solve & Share:** Consider asking students to solve this on a blank page, white board, etc. so that the task offers multiple entry points. This will also offer you more information on how your students are understanding and seeing multiplicative situations. After student solution methods and reasoning have been shared, asking students questions such as, "What do all the products have in common?" (e.g. they're even) and "Why is that?" will help them to make generalizations and reach the understandings stated in *Beginning of the Big Idea* for when 2 is a factor. Have students confirm, clarify, or correct their ideas during the *Visual Learning Animation*. **-continues next page-**

	place. When 5 is multiplied by an even number, it has a 0 in the ones place.	**Visual Learning:** The *Visual Learning Animation* for this topic is helpful for reinforcing understanding of meanings of multiplication. If students are still struggling to understand multiplication as the joining of equal groups, view and discuss the ideas modeled as a class. Building in additional pausing points during the *Visual Learning Animation* will allow students to practice the skip counts and represent them on an open number line. **Convince Me:** Consider having a whole class discussion around the *Convince Me!* so students can continue their discovery of patterns in products when 2 and/or 5 is a factor. **Assess and Differentiate/Intervention Activity:** If time permits, teach students how to play "Quick Questions" (TE, p. 65A). All students should have the opportunity to play games that provide opportunities for practicing strategies for facts with 2 and 5 as factors. Students may also continue to play any of the games from topic 1. Consider utilizing the following question formats during practice: Example 1 Full Statement **Example Stem 1:** There are 3 rows of pictures with 2 pictures in each row. How many pictures are there? Enter your answer in the response box. Full Statement **Example Stem 2:** The pictures on a page in a picture album are in 3 rows and 2 columns. How many pictures are on the page? Enter your answer in the response box.
Lesson 2-3: Apply Properties: Multiply by 0 and 1		
3.OA.A.3 **3.OA.A.1** **3.OA.D.9** MP.1 MP.2 MP.3 MP.7	**Access Prior Learning:** In Topic 1, students developed an understanding of multiplication as joining equal groups to find the total number of objects in groups. In 2nd grade, students were secure in their understanding of the Zero Property of Addition. **Developing the Big Idea:** Students *develop* the understanding of multiplication as joining equal groups of objects to *begin* to build an understanding of the Zero Property of Multiplication and Identity (One) Property of Multiplication.	**Instructional note:** See the Instruction note at the top of this document for an explanation of ***moving lesson 2-2.*** **Solve & Share:** For concrete learners or students grappling with the misconception that multiplication always makes numbers bigger, you may consider having paper plates or bags available so that students can model having 6 groups of 0 objects. To support students' development of MP. 4 Model with mathematics, you might consider asking students how they could model or show this problem before letting them work on the *Solve & Share*. Before moving onto the *Visual Learning*, ask students "If Carlos had 6 bags with 1 apple in each bag, how many apples would he have?" Continue to question students to develop a class conjecture about multiplying by 1. Confirm, clarify, and correct this conjecture during the *Visual Learning Animation*. **Visual Learning:** Consider pausing the *Visual Learning Animation* after they introduce each property to have students test the property using counters and groups of other factors to confirm the stated property. **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 26 as this problem allows for multiple entry points and answers. When a student solves the problem, you can extend thinking by asking if that is the only answer. Students may give a generalized rule for the answer (any number greater than 4, as 4 is the minimum number needed to have a greater number of bikes than Barb's class). **Assess and Differentiate/Intervention Activity:** If time permits, you may consider replacing *Problem Solving Reading Mat* with either the games from previous topics, the game *Quick Questions* (TE, p. 65A), or the *Fluency Practice Activity* (TE, p. 97). Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p.77A).

Lesson 2-4: Multiply by 10		
3.OA.A.3 **3.OA.A.1** **3.OA.D.9** MP.2 MP.3 **MP.4** MP.7 MP.8	**Access Prior Learning:** Students have been skip counting by 10 since Kindergarten. In Topic 1 students used skip counting on the open number line to represent multiplication. **Developing the Big Idea:** Students are *developing* their understanding of patterns in multiplication by identifying a pattern when multiplying by 10.	**Solve & Share:** Consider providing students with tools, such as two-colored counters or place-value blocks to solve the problem. Students may only write the products for each week which can make identifying a pattern for multiplying by 10 difficult to see. In this event, consider asking students to write out the equations they used to solve each week and to looks for patterns in the factors and products. During the class discussion of students' solution methods and reasoning, push to get students to use place value reasoning to support their explanations of the justification offered in the *Transition to the Visual Learning Bridge* (TE, p. 79). For example, students may explain, "Since 6 x 10 means we have 6 groups of 10 miles we do not have any ones because 6 tens is 60 resulting in a 0 in the ones place for all products when 10 is a factor." Referring to 60, 70, and 80 as multiples of 10 will support students' understanding of the term multiples. **Visual Learning:** Consider pausing the video after it asks, "How many miles will Greg run to train for the race?" Discuss as a class what operation is needed to solve this question. The video will ask students this question as it's showing the 10's times tables; however, at this point connect student responses to the 10s fact table. This will support student understanding of multiplication being 'groups of'. Avoid teaching students the "zero trick" of just adding a zero to the right and instead maintain the focus on patterns that appear when multiplying by 10. Consider continuing to support students' understanding of the term "multiple(s)" by asking them to identify the multiples of 10 in the 10's times table. **Independent Practice/Math Practices and Problem Solving:** Consider including item 15 so that students have the opportunity to revisit reasoning with repeated subtraction situations. **Assess and Differentiate:** The *On-Level* and *Advanced Activity Centers* for this lesson includes 9 as a factor. Consider allowing students to play the game, but first challenge them to use what they know about multiplication with factors of 10 to develop a strategy for solving problems with 9 as a factor. Alternatively, you may wish to have students play a game from previous topics or lessons. Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p.83A). ***CTC: *Math Practices and Problem Solving*** (student work samples)
Lesson 2-2: 9 as a Factor		
3.OA.A.3 **3.OA.A.1** **3.OA.D.9** MP.1 MP.2 MP.3 MP.7	**Access Prior Learning:** In Topic 1 students came to understand multiplication as the joining together of equal groups. In *Lesson 2-4, Multiples of 10*, students identified a pattern for solving for multiplication problems with 10 as a factor. **Developing the Big Idea:** In this lesson students are *developing* their understanding of multiplication as joining equal groups and of multiples of 10 to generate a derived fact strategy for multiplication problems with 9 as a factor.	The purpose of this lesson is to explore the patterns that occur in multiplication and build mathematics curiosity and wonder at why these patterns occur. 9 as a factor reveals fascinating and unique opportunities to recognize and explore patterns beyond those that are found when looking at other factors, which may just reveal multiples or simple and easily recognizable patterns. Exploring 9 as a factor can push toward arithmetic patterns beyond those explored in earlier grades. The pattern explorations should not be seen as tricks to help students memorize the 9's facts. If using 2-2 after 2-4 then build opportunities for students to use **known** facts such as 10 x 4 (40) to **derive** a 9 fact such as 9 x 4 (I know that 10 groups of 4 is 40 so *one less group* of 4 will be 36). Moving from known to derived facts will be explored further in topic 3. **Solve & Share:** Prior to the *Solve & Share* assess student readiness by asking students to state the meaning of the equation 6 x 10 from yesterday's *Solve & Share* (e.g. 6 x 10 means we have 6 groups of 10 for a total of 60) and asking what if Duke ran 9 miles. How many will he run in 6 weeks? Consider asking a student that has direct-modeled the *Solve & Share* either with counters or pictorial representations *and* a student that used a 10 as a factor and completed a derived fact strategy to share. **-continues next page-**

Orchestrate a class discussion around these two solution methods and the reasoning used. As a class, consider how to model the math (MP.4) when using 10 as a factor. Connect the reasoning between the student's model who use a derived fact to model and that of the student that chose to direct model the 4 x 9 = 36

Look Back:
Consider discussing the *Look Back!* problem revisit ideas about the Commutative (Order) Property of Multiplication.

Visual Learning:
Since the *Visual Learning Animation* is more procedural than conceptual, consider replacing the animation by writing the 10's and 9's times tables next to each other. Facilitate a discussion to help students identify a pattern for using multiples of 10 to solve problems with 9 as a factor. Students can generalize to discover a derived fact strategy for multiplication problems with 9 as a factor. For example:

1 x 10 = 10	1 x 9 = 9
2 x 10 = 20	2 x 9 = 18
3 x 10 = 30	3 x 9 = 27
4 x 10 = 40	4 x 9 = 36
5 x 10 = 50	5 x 9 = 45
6 x 10 = 60	6 x 9 = 54
7 x 10 = 70	7 x 9 = 63
8 x 10 = 80	8 x 9 = 72
9 x 10 = 90	9 x 9 = 81
10 x 10 = 100	10 x 9 = 90

See the *Instructional Note* at the beginning of this topic for an explanation of student reasoning of this strategy.

Consider modeling the multiple by 10 and subtract the extra group using Base-10 blocks or counters and connect to the array model to support student understanding.

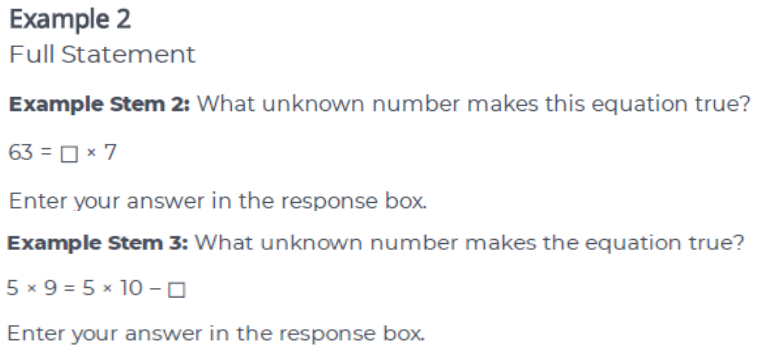
If the *Visual Learning Animation* is replaced with the above activity, question 2 in the *Guided Practice* will need to be skipped or reworded to have students describe using a multiple of 10 to solve a 9s fact.

Independent Practice/Math Practices and Problem Solving:
Notice that item 15 is the same equation students used in today's *Solve & Share*. This is an opportunity to see if students will recognize that they have worked with this problem already and it will therefore have the same product.

Assess and Differentiate/Intervention Activity:
If time permits, teach students how to play *Toss and Talk* (TE, p. 83A). All students should have the opportunity to play this game.

Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p.71A). For this activity, you can still do the *Intervention Activity Modeling 9s Facts*; however, consider replacing the worksheet with relating the models made to the finger strategy for finding multiples of 9.

Consider utilizing the following question format during practice:



Lesson 2-5: Multiplication Facts: 0, 1, 2, 5, 9, and 10		
3.OA.A.3 **3.OA.A.1** **3.OA.D.9** MP.3 **MP.4** MP.5 MP.6 MP.7	**Access Prior Learning:** In previous lessons in Topic 2 students have identified patterns that can be used as strategies for solving for facts with 0, 1, 2, 5, 9, and 10. **Developing the Big Idea:** This lesson further *develops* the understandings students began to understand in the previous lessons to apply the strategies for solving for facts with 0, 1, 2, 5, 9, and 10 as a factor.	**Solve & Share:** For the whole class discussion consider sequencing solution strategies so that the first student to share has a solution method that is similar to Stephanie's work (TE, p. 85). Consider having the second student to share be one that used solutions from having solved for the previous numbers of boxes to determine the products for the other boxes. **Visual Learning:** As students likely did not use a bar diagram to find the solution to the *Solve & Share*, the *Try It!* provides the opportunity to work with modeling the math (MP.4) using a bar diagram. It may be helpful to discuss how the bar diagram is able to represent the joining of the equal groups. **Assess and Differentiate:** Child-watch to identify students who need additional support and pull them into a small group to complete the Intervention Activity (TE, p.23A). ***CTC: *Quick Check*** (digital platform) Consider utilizing the following question formats during practice: Example 4 Full Statement **Example Stem 4:** Claire arranges 6 pictures into an array with 3 rows. How many columns of pictures are in the array? Enter your answer in the response box. Example 6 Full Statement **Example Stem 6:** Lisa arranges 6 pictures into an array with 2 columns. How many rows of pictures are in the array? Enter your answer in the response box.
Lesson 2-6: Math Practices and Problem Solving- Model with Math		
3.OA.A.3 **MP.4** MP.1 MP.2 MP.3 MP.5	**Access Prior Learning:** In this topic students have identified patterns that can be used as strategies for solving for facts with 0, 1, 2, 5, 9, and 10. Lesson 1-7 involved students solving for a 2-step word problem. Students also worked with 2-step word problems involving addition and subtraction and the idea of a "hidden problem" in 2nd grade Topics 8, 13 & 14. **Developing the Big Idea:** In this lesson, students are *developing* their understanding of multiplication as joining equal groups and the use of patterns for multiplying with 0, 1, 2, 5, 9, and 10.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 4. Refer to the Math Practices and *Problem Solving Handbook* (TE p. F24-F24A, F29) for suggestions on how to develop, connect and assess this Math Practice. Also, reference the handbook in the student edition (SE, p. F24). **Solve & Share:** Consider reintroducing MP. 4 Thinking Habits (SE, p. F24) before introducing the *Solve & Share*. Restating that an equation is an example of MP.4 Modeling the Math can be a good reminder. Many students are under the misconception that MP. 4 means they must show a drawing or concrete representation of the math. While having a drawn or concrete representation of the math can make for a stronger argument (MP.3), it is not necessary for modeling mathematical situations. Consider using the time when students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.4 that are listed in the *Math Practices and Problem Solving Handbook* (p. F24A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice. **Convince Me:** Consider assigning the *Convince Me!* as it offers another opportunity to work with MP.4 and assess for behaviors attributed to this math practice. **Assess and Differentiate:** If time permits, consider assigning the *Math and Science Activity* (TE, p. 95A) as this relates the mathematics in this topic to a real world context. Child-watch to identify students who need additional support and pull them into a small group to complete the Intervention Activity (TE, p.95A).

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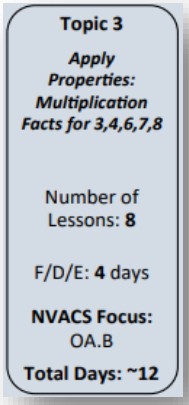
Grade 3 Topic 3: Apply Properties: Multiplication Facts for 3, 4, 6, 7, 8



Big Conceptual Idea: Operations and Algebraic Thinking, K-5 (pp. 22-28)

Prior to instruction, view the Topic 3 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 105A-105F), the Topic Planner (pp.105I-105K), all 8 lessons, and the Topic Performance Assessment (pp. 163-164A).

Mathematical Background: Read Topic 3-4 Cluster Overview/Math Background (pp. 105A-105F)	**Topic Essential Question:** How can unknown multiplication facts be found using known facts? *Reference Answering the Topic Essential Question (TE, pp. 161-162) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

3-1	3-2	3-3	3-4	3-5	3-6	3-8	3-7	Assessment

4 F/D/E days used strategically throughout the topic

3rd Grade Curriculum Pacing Framework: Balanced Calendar

Instructional Note:

The Nevada Academic Content Standard (NVACS) cluster 3.OA.B states, "Understand properties of multiplication and the relationship between multiplication and division". In Topic 3, students work with the Properties of Operations to build reasoning strategies using known facts explored in Topic 2. Students do this by multiplying with 3, 4, 6, 7, and 8 as factors. Students will continue to work with multiplication using contextual problems (3.OA.A.3) and explore patterns that occur with factors and products (3.OA.D.9). Building an understanding of the Distributive and Associative Properties and how to reason with known facts will lead to a strong conceptual understanding of multiplication that is the foundation for fluency. Students continue this work throughout several topics (See Topic 5).

The *Topic 3 Professional Development Video* states, "Using known facts along with the properties of multiplication is a strategy for learning the multiplication facts for 3, 4, 6, 7, and 8" (Dr. Schielack, **enVision**math**2.0**, 2016). This topic introduces the Distributive and Associative Properties of Multiplication with a focus on the standard 3.OA.B.5, "Apply properties of operations as strategies to multiply and divide." (NVACS, p. 23). While grouping symbols are not explicitly stated in the standards until 5th grade, the use of parentheses is an assumed part of the mathematics in the properties and should be used to communicate the grouping of the expressions and thus the order of the operations. For more information about the use of parentheses in 3rd grade and the progression of Order of Operations please read page 27 of the K-5, Operations and Algebraic Thinking progression document. The footnote on this standard indicates that students do not need to use the formal terms for these properties; therefore, it is acceptable for students to refer to them as the turn-around, break-apart, and order properties of multiplication. However, you may want to consider restating their informal language with the formal terms to support precise mathematical vocabulary development.

Topic 3 uses the Distributive Property of Multiplication extensively to support student understanding by linking an array with the decomposition of a factor. Students model decomposing a factor into smaller factors to breaking apart larger arrays into smaller arrays. For example, given an array that models 7 x 5, students may choose to decompose or break the first factor (7) into a 5 and 2 as they know their 5 facts and their 2 facts. Thus, students are using the distributive property to solve unknown facts using known facts: 7 x 5 = (5 x 5) + (2 x 5).

In lesson 3-7, the focus is on understanding the Associative Property, which allows factors to be grouped in different ways. When given three or more factors, students group the factors differently depending upon what is more efficient for them. For example, given the factors 2 x 5 x 3 (which can be modeled with two separate 5 x 3 arrays), students may group the factors as (2 x 5) x 3 or as 2 x (5 x 3). They should see the equivalence between these groupings and the new facts created by associating different factors together; in this case 10 x 3 or 2 x 15. Consider using one of the A/D/E days for this topic to spend more time exploring these ideas from the Associative Property of Multiplication.

When students demonstrate readiness, consider replacing the two-colored counters used in arrays with colored tiles to begin building area models. This will begin to lay the foundation for the connection between the array model and the area model explored in Topic 6. Note: If colored tiles are used, be sure that there are no gaps between tiles as you are now connecting to area concepts.

Looking ahead to the Topic Assessment, consider having tools available for students that may need them. Item 11 Part B requires students to generalize their understanding and apply it to a new situation. In the Topic Performance Assessment for item 2 Part B, accept multiple answers for where students draw the line; also accept responses where students have drawn multiple lines to use the Associative Property of Multiplication.

Focus Math Practice 8: Look for and express regularity in repeated reasoning
Focus on opportunities for students to develop Mathematical Practice 8 behaviors, as this is the focus of the Math Practices and Problem Solving lesson 3-8. Reference the Teacher's Edition (TE, pp. F28 - F28A) and the Nevada Academic Content Standards for Mathematical Practice (2010, p. 8).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
Distributive (break-apart) Property of Multiplication Associative (Grouping) Property of Multiplication	*factor* *product* *commutative property of multiplication* *doubles* *halving*

Additional terminology that students may need support with: *break-apart, addend, sum, compose, decompose, generalization*

***Collaborative Team Conversations (CTC)**
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students using the properties of multiplication and known facts to find products of unknown facts?"

Lesson	Evidence	Look for
3-1	***Solve & Share*** (student work samples)	Focus CTC around the big idea: • students understand that arrays can be broken apart (decomposed) into smaller arrays based on the distributive property.
3-3	***Math Practices and Problem Solving*** (student work samples) Items 22 and 23	Focus CTC around the big idea: • students apply the properties of multiplication with 4 as a factor. • students use known facts to find products of unknown facts.

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 161-164	Use *Scoring Guide* TE pp. 161-164A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	**Mathematical Development of the Big Idea**	**Instructional Clarifications & Considerations**
Lesson 3-1: The Distributive Property		
3.OA.B.5 MP.2 MP.3 MP.4 MP.6 MP.7	**Access Prior Learning:** Commutative Property of Multiplication, familiarity with facts that include 2, 5, 0, 1, 10, and 9 as a factor. **Developing the Big Idea:** Students are *beginning* to understand that the Distributive Property of Multiplication can be used to break a large array into a small array that represents known facts.	**Topic Opener:** Introduce the *Topic Essential Question*, "How can unknown multiplication facts be found using known facts?" (TE, p. 105). Consider making this an anchor chart in your classroom where each day new ideas are added so that students can see the development and make connections throughout the topic. If building an anchor chart, this could be an appropriate time to quickly revisit the Commutative Property of Multiplication and add it to the chart. You may want to expand on this idea and include in the chart that since they already know 2 x 7 = 14 they also know that 7 x 2 = 14. You might also consider having students complete the *Review What You Know* prior to beginning instruction on Topic 3 so that you can respond to student instructional needs using the *Item Analysis for Diagnosis and Intervention* (TE, p. 106-108). Consider introducing vocabulary as they encounter them in the lessons rather than introducing all terms at the beginning of the lesson. **Solve & Share:** Consider waiting to distribute the 25 two-color counters until students suggest them in response to the question, "What tool can you use to solve this problem?" (TE, p. 109). ***-continues on next page-***

		While students are working on the Solve & Share consider asking them: • "What equation does the original array represent?" (e.g. 5 x 4) • "What equation(s) does the broken array represent?" • "Why did you choose to break it up like that?" In these questions, you are starting to lay the foundations for students to understand that breaking up the array using known facts to find the product is one way to make simpler problems. Consider returning to these questions in the whole group discussion of students' solutions and reasoning. **Look Back:** Consider discussing as a class the *Look Back!* question to help students develop an understanding of how the distributive property helps us to use known facts to solve for unknown facts (called derived facts). **Visual Learning:** Before clicking on *Try It!* consider having groups make the different possible arrays presented in the *Try It!* and do a quick Gallery Walk of the different ways to break-up the 7 x 4 array, taking notice of the known facts in each representation. When watching the video and formalizing the definition of the Distributive Property of Multiplication, consider taking a moment to discuss the meaning of, "the sum of two other facts". It is easy for students to overlook that in the Distributive Property of Multiplication we are adding the products of our two partial products. Students will also often miss that we are adding the products because we broke one of the factors into addends of the factor. **Independent Practice/Math Practices and Problem Solving:** The *Quick Check* asks students to demonstrate understanding of equations representing the Distributive Property of Multiplication. Teachers might find that this needs to be a 2-day lesson. In that case, consider using the "Higher Order Thinking" item 16 as a *Solve & Share*, making sure to provide counters, and connect student representations to the conventions of writing the equation. You may consider using the *Another Look!* video as the *Visual Learning* animation. Lessons 3-2 through 3-6 provide students with the opportunity to gain security in writing equations showing the use of the Distributive Property of Multiplication. Finally, should you choose to make this a 2-day lesson, consider building in time for students to practice building the known fact arrays by giving student pairs a multiplication fact to model with the counters the different factor pairs they can make and then writing the equations. For example, given 6 x 7 students could create broken arrays that show: 6 x 7 = (5 x 7) + (1 x 7); 6 x 7 = (3 x 7) + (3 x 7); etc.
Lesson 3-2: Apply Properties: 3 as a Factor		
3.OA.B.5 **3.OA.A.3** **3.OA.D.9** MP.3 MP.5 MP.7 **MP.8**	**Access Prior Learning:** In Topic 2, Grade 3 students identified the patterns in multiplying with 2 and 1 as a factor. **Developing the Big Idea:** Students further *develop* their understanding of multiplying by 2 and 1 as a factor and the Distributive Property of Multiplication to solve for unknown multiplication problems with 3 as a factor.	**Solve & Share:** Watch for students that choose to solve this problem using repeated addition. Encourage these students to write the represented multiplication problem and then use what they learned about the Distributive Property of Multiplication to help them solve. You may need to ask these students, "Are there known facts you can use to break-up the array to solve this problem?" Attempt discovering the mathematical understanding described in *Transition to the Visual Learning Bridge* (TE, p. 115) during the whole class discussion from student solutions and reasoning. **Look Back:** Consider discussing the *Look Back!* as it supports students' developing understanding of MP.*8 Look for and express regularity in repeated reasoning*. Students continue to work with the Distributive Property of Multiplication to break-apart unknown facts into known facts for solving. **Convince Me:** Consider discussing the *Convince Me!* as a class to check for understanding. In the discussion, make sure that students understand that the 3 is decomposed into 2 and 1 because we know facts for 2s and 1s. **Assess & Differentiate:** If time permits, teach students how to play *Toss and Talk* (TE, p. 119A). All students should have the opportunity to play this game.

Lesson 3-3: Apply Properties: 4 as a Factor

3.OA.B.5
3.OA.A.3
3.OA.D.9

MP.1
MP.2
MP.3
MP.7

Access Prior Learning:
In Topic 2, Grade 3 students identified the patterns in multiplying with 2 and 1 as a factor. In lesson 3-2, students used 1s and 2s to solve for problems with 3 as a factor.

Developing the Big Idea:
Students further *develop* their understanding of multiplication problems with 1, 2, and 3 as a factor to solve for multiplication facts with 4 as a factor. Students extend their understanding of the Distributive Property of Multiplication to generate the understanding that the Distributive Property of Multiplication can be used to break the 4s multiplication facts into smaller known facts.

Solve & Share:
To assess students' readiness, consider asking students how we could decompose 4 using addition and multiplication. Focus responses on 2 + 2 and 1 + 3 for addition and 2 x 2 for multiplication as these provide factors for making known facts from multiplication problems with 4 as a factor.

Look Back:
Depending upon student readiness you may consider discussing the *Look Back!* question to begin developing understanding of the Associative Property of Multiplication. After reaching the understanding described for the question (TE, p. 121) consider asking students what multiplication fact works with doubles (e.g. 2s). If you feel students are ready and it wouldn't further confuse them, you can then connect the equation (2 x 8) + (2 x 8) = 32 to having (2 x 8) x 2 = 32 because we have 2 x 8 doubled.

The *Visual Learning Animation* will connect the idea that 2's facts are doubles and since 4 is a double of 2, we can think of 4 facts as "double-double." It does not, however, show it in equation form. For more information on decomposing using the Distributive Property read page 26 of the K-5, Operations and Algebraic Thinking. Additionally, more information on this can be found in the *Topic Professional Development Video*.

Visual Learning:
If students are struggling with understanding how to break-apart a large array (unknown fact) into known facts, smaller arrays, it may be more helpful to have students build the array and break it apart rather than doing the *Try It!* digitally.

Convince Me:
If you did not do the *Look Back!* as described above, but now feel students are ready to explore those ideas, the *Convince Me!* provides this opportunity.

Independent Practice/Math Practices and Problem Solving:
Item 19 provides students the opportunity to reason with a 2-step story problem where the first step requires multiplication and the second step is an additive compare situation. The reasoning students will have to engage in to solve this problem will also help prepare students for the reasoning necessary should you choose to do the Topic Performance Assessment.

Assess and Differentiate/Intervention Activity:
If time permits, you may consider replacing the *Math and Science Activity* with a game that will provide an opportunity for meaningful practice with any of the factors that students have already developed strategies.

Consider utilizing the following question format during practice:

Example 1
Full Statement

Example Stem 1: Decide if each expression is equal to 5 × 9. Select Yes or No for each expression.

	Yes	No
5 × (5 + 4)		
(5 × 5) + 4		
(5 × 5) + (5 × 4)		

Lesson 3-4: Apply Properties: 6 and 7 as a Factor

3.OA.B.5
3.OA.A.3
3.OA.D.9

MP.1
MP.4
MP.5
MP.7

Access Prior Learning:
In Topic 2, Grade 3 students identified the patterns in multiplying with 1, 2, and 5 as a factor. In *Lesson 3-3,* students *developed* the understanding that a strategy for 4's facts is to think of it as "double-double."

Developing the Big Idea:
In this lesson, students further *develop* their understanding of multiplication problems with 1, 2, 3, 4, and 5 as a factor to solve for

(Possible 2-day lesson)

Instructional note: Determining if this lesson needs to be a 2-day lesson should be based on child-watching and formative data from previous lessons. If students are still struggling with using 3, 4, or 5-fact as their "known fact" then they may benefit from making this a 2-day lesson.

Students solving for a 6-fact may use a 5 and 1-fact **or** a 3-fact they double as their *known facts*. When solving for a 7-fact, students may use a 5 and 2-fact **or** a 4 and 3-fact. In the event it is determined that students would benefit from a 2-day lesson consider the following structure:
Day 1: Solve and Share as is and the Visual Learning
Day 2: Rewrite the Solve and Share to have 7 rows with 7 chairs in each. Follow with the Another Look! video

-continues next page-

	multiplication facts with 6 or 7 as a factor. Students also further *develop* their understanding of "double-double" to 6 facts.	**Solve & Share:** Consider discussing the *Look Back!* problem and look for a student that explains the rule as the one provided for the prompt in the teacher's edition. Also look for a student that explains it by decomposing 6 and then uses the Associative Property of Multiplication (e.g. (2 x 6) x 3 or 2 x (6 x 3)). For students that use the latter equation you might consider posing the question, "What property of multiplication allows us to switch to solving 6 x 3 first?" Then have both students share their reasoning. While the Associative Property of Multiplication is not an outcome for this lesson if you have students already using this reasoning it may be helpful to start forming these ideas now. **Visual Learning:** The *Visual Learning Animation* makes one strategy explicit for solving for a 6 fact. This strategy will also work for 7 facts but the animation does not make this connection. Therefore, you may want to ask students how they could use this strategy to solve for 7's facts. Alternatively, you may consider replacing the *Visual Learning* animation with the *Another Look!* video as it makes the strategy explicit for both 6 & 7's facts. If you go this route, consider pausing after it displays the equations "6 x 4 = ?" to collect student responses on how they might solve using known facts (e.g. double-double with the four so (6 x 2) + (6 x 2) = 24, double-double with the six so (3 x 4) + (3 x 4), or using 2s facts so (2 x 4) + (2 x 4) + (2 x 4) = 24, using 3s facts so (6 x 3) + (6 x 1) = 24). Consider asking students if doubling will work for 7 (e.g. no because 7 is not a double). You may want to build in additional class practice with counters in modeling and writing the equations for these strategies. **Assess and Differentiate/Intervention Activity:** If time permits, teach students how to play *Teamwork* (TE p. 131A). All students should have the opportunity to play this game as this reinforces the idea of using known facts to solve for unknown facts and provides meaningful practice with identifying the known fact and working from that fact. You might consider modifying to other factors (e.g. have students role dice to get the factors) for future extended play. Consider utilizing the following question formats during practice: **Example Stem 5:** What unknown number makes the equation true? $6 \times 6 = 6 \times 5 + \square$ Enter your answer in the response box.
Lesson 3-5: Apply Properties: 8 as a Factor		
3.OA.B.5 **3.OA.A.3** **3.OA.D.9** MP.1 MP.4 MP.7 **MP.8**	**Access Prior Learning:** In Topic 2, Grade 3 students identified the patterns in multiplying with 1, 2, and 5 as a factor. In previous lessons within Topic 3, students have *developed* the understanding that they can use these known facts to solve for facts with 3, 4, 6, and 7 as a factor. **Developing the Big Idea:** This lesson further *develops* the idea that we can use known facts to solve for unknown facts using the Distributive Property of Multiplication.	**Solve & Share:** Consider assigning the *Look Back!* prompt and asking groups to take a different factor to share out. These are good ideas to add to the class anchor chart. **Visual Learning:** Students that are struggling to keep track of all the doubles shown in the *Visual Learning Animation* may prefer to use the Distributive Property of Multiplication by decomposing the 8 into 5 + 3. This also might be a good time to revisit the Commutative Property of Multiplication by posing the question, "For 8 x 2, what is the most appropriate/efficient strategy for me to solve (e.g. I know 2 x 8 = 16 so it's 16)?" **Convince Me:** You might consider discussing the *Convince Me!* whole group. **Independent Practice/Math Practices and Problem Solving:** For more information beyond the explanation provided in the Teacher's Edition on *Quick Check* item 19, watch the *Listen and Look For* video for this lesson. **Assess and Differentiate/Intervention Activity:** If time permits, you may consider replacing the Math and Science Activity with the game *Teamwork* (TE, p. 131A). Please see the comments for this game in Lesson 3-4. Consider utilizing the following question format during practice: **Example Stem 6:** What unknown number makes the equation true? $8 \times 7 = 5 \times 7 + \square \times 7$ Enter your answer in the response box. **-continues next page-**

Lesson 3-6: Practice Multiplication Facts

3.OA.B.5
3.OA.A.3

MP.1
MP.2
MP.3
MP.4

Access Prior Learning:
In Topic 2, Grade 3 students identified the patterns in multiplying with 1, 2, and 5 as a factor. In previous lessons in this topic students have *developed* the understanding that they can use these known facts to solve for facts with 3, 4, 6, 7, and 8 as a factor.

Developing the Big Idea:
This lesson further *develops* the understanding that we can use known facts to solve for unknown facts by including using a bar diagram as a model for the math.

Solve & Share:
To assess student readiness, you may consider posing the question, "How can we model multiplication using a bar diagram? What other ways can we model multiplication?"

Visual Learning:
Consider pausing the video after it shows, "Each section is 3 feet long" and posing the question, "What is our multiplication equation for this problem? (e.g. 9 x 3 =?) How could you solve this?" Provide time for students to solve and use this as an opportunity to see what reasoning strategies students are using to solve.

Convince Me:
If you already provided time for students to solve for 9 x 3, you might consider doing the *Convince Me!* to have students share their strategies and reasoning that wasn't shown in the video. Having exposure to these strategies would be beneficial for the whole group.

Independent Practice/Math Practices and Problem Solving:
Students have to reason that a week has 7 days in order to solve item 26; therefore, it might be beneficial to assign this item to build problem solving reasoning habits.

Assess and Differentiate:
If time permits, you may consider replacing the *Problem Solving Read Mat* with the game *Teamwork* (TE, p. 131A). Please see the comments for this game in Lesson 3-4.

Child-watch to identify students who need additional support and pull them into a small group to do the Intervention Activity (TE, p.23A).

Consider utilizing the following question format during practice:
Example Stem 2: What unknown number makes the equation true?

$8 \times 6 = 8 \times \square \times 2$

Enter your answer in the response box.

Lesson 3-8: Math Practices and Problem Solving- Repeated Reasoning

MP.8
MP.1
MP.3
MP.5
MP.7

3.OA.B.5
3.OA.A.3

Access Prior Learning:
In this topic students have developed understanding of how they can use known facts for 1, 2, and 5 to solve for unknown facts for 3, 4, 5, 6, 7, 8, and 9 focusing mostly on the Distributive Property of Multiplication as a justification for why this works.

Developing the Big Idea:
In this lesson, students *develop* their understanding of MP.8 Use repeated reasoning to *secure* their understanding of using the Distributive Property of Multiplication as a strategy for solving unknown facts.

This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 9. Refer to the *Math Practices and Problem Solving Handbook* (TE p. F28-F28A, F29) for suggestions on how to develop, connect and assess this Math Practice. Also reference the handbook in the student edition (SE, p. F28).

Solve & Share:
Consider reintroducing MP. 8 *Look for and express regularity in repeated reasoning* Thinking Habits (SE, p. F28) before introducing the *Solve & Share*. You may want to restate that an equation is an example of MP. 8 *Look for and express regularity in repeated reasoning*. In this case the general method we are wanting students to notice is the use of the Distributive Property of Multiplication in solving for unknown facts.

You may also consider using the time where students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.8 that are listed in the *Math Practices and Problem Solving Handbook* (F28A), and afterwards discussing student solution methods and reasoning. Ask students to self-score for the behaviors associated with this math practice.

Finally, during the whole group discussion on students' solution strategies and reasoning, ensure that the generalization recognizes that when decomposing a factor, it's being broken into addends. For further ideas on how to facilitate the conversation so that students recognize this, preview the *Listen and Look For* video prior to teaching the lesson so that you can use their questions. This understanding is important as students learn to decompose factors and make known facts from unknown facts.

Visual Learning:
During the *Visual Learning Animation*, consider asking the students what they decomposed the factors for each equation into to make a known fact (e.g. A 3 was decomposed into the addends 2 and 1, B 4 was decomposed into the addends 2 & 2, C 6 was decomposed into the addends 5 and 1, D 7 was decomposed into the addends 5 and 2).

Independent Practice/Math Practices and Problem Solving:
Watch for students that do not recognize the "35 minutes to bake the pizzas" is extraneous information and try to use it to solve the problem. These students are not reasoning with the context to make sense with the mathematics and need additional support on how to problem solve.

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		Assess and Differentiate/Intervention Activity: If time permits, teach students how to play *Clip and Cover* (TE, p. 155A). All students should have the opportunity to play the games. Child-watch to identify students who need additional support and pull them into a small group to do the Intervention Activity (TE, p.155A).
Lesson 3-7: The Associative Property: Multiply by 3 Factors		
3.OA.B.5 **3.OA.A.3** MP.1 MP.2 MP.3 MP.4 **MP.8**	**Access Prior Learning:** In Topic 2, Grade 3 students identified the patterns in multiplying with 1, 2, and 5 as a factor. In previous lessons in this topic, students have *developed* the understanding of using the Distributive Property of Multiplication to use known facts to solve for facts with 3, 4, 6, 7, and 8 as a factor. **Developing the Big Idea:** This lesson extends the understandings they developed in the previous lessons to apply the strategies for solving for facts with 0, 1, 2, 5, 9, and 10.	Watching the *Topic Professional Development Video* will help to clarify the ideas around the Associative Property of Multiplication Prior and strengthen facilitation of the *Visual Learning Animation* during instruction. **Solve & Share:** The *Listen and Look For* video for this lesson goes into great detail about how to use this problem to facilitate understanding of the associative property of multiplication. While students work to solve this problem, watch for those that do not include parentheses as a grouping symbol. These students might be working with the misconception that parentheses are limited to the Distributive Property of Multiplication. Offer the clarification that parentheses are used anytime we want to communicate that we've grouped expressions together. Watch for students that represent the problem as (5 x 3) + (5 x 3) or 15 + 15. Help them connect these forms of repeated addition to multiplication; ask students to write a number sentence for this situation using only multiplication. 2 x 15 could be a first step. Where did the 15 come from? Do students notice it represents the total number of squares in one quilt? Can they replace the 15 with (5 x 3) to create the number sentence 2 x (5 x 3)? Help students to see the equivalence between these different forms. What will happen now if we decide to use the 2 x (5 x 3) model but instead associate the factors 2 and 5? We have created a new number sentence, (2 x 5) x 3 and a different number sentence of 10 x 3. Do students notice that there are 10 rows between the two quilts with 3 squares in each row? The associative property allows us to represent the same situation using different but equivalent number sentence models. **Visual Learning:** The *Visual Learning Animation* uses a very similar problem to model use of the associative property of multiplication. Help students make connections between their reasoning and strategies used during the *Solve and Share* and those seen in the *Visual Learning Animation*. In this lesson students have seen that when working with 3 factors, the order they multiply them does not change the final product although it does require an extra step. Connect to our *Topic Essential Question*, "How can unknown multiplication facts be found using known facts?" by asking, "How can we use the Associative Property of Multiplication to decompose an unknown multiplication fact into known facts? Can we solve 4 x 2 x 5 as 8 x 5 or 4 x 10? How can we use this property to make simpler problems for solving?" **Assess and Differentiate:** If time permits, you may consider returning to the game *Teamwork* (TE p.131A) with a variety of factors (see notes in Lesson 3-4) and have students decomposing the larger arrays into 2 or more small arrays while writing the multiplication equation modeled.

References

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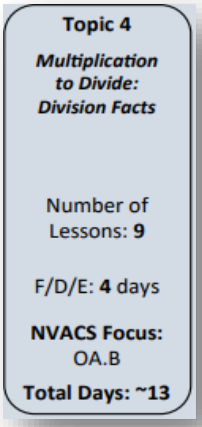
Grade 3 Topic 4: Use Multiplication to Divide-Division Facts



3rd Grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Operations and Algebraic Thinking (pp. 22-28)

Prior to instruction, view the Topic 4 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 105A-105F), the Topic Planner (pp.165I-165D), all 9 lessons, and the Topic Performance Assessment (pp. 233-234A).

Mathematical Background: Read Topic 4 Cluster Overview/Math Background (TE, pp. 105A-105F)	**Topic Essential Question:** How can unknown division facts be found using known multiplication facts? *Reference Answering the Topic Essential Question (TE, pp. 229-230) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

4-1	4-2	4-3	4-4	4-5	4-6	4-7	4-8	4-9	Assessment

4 F/D/E days used strategically throughout the topic

Instructional note:

This topic focuses on understanding the inverse relationship of multiplication and division and using multiplication to solve division problems. These understandings meet the 2010 Nevada Academic Content Standards (NVACS) 3.OA.B6, "Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8*." Students that understand this inverse relationship realize that they already know division facts because they know the multiplication facts.

Although students explored the concept of division in Topic 1 as fair sharing and repeated subtraction, they have not yet fully explored the relationship between multiplication and division. Students are familiar with using models such as arrays and bar diagrams to represent multiplication situations. Now they will be asked to use these models to explore division situations. For example, given a visual model for the problem 24 ÷ 6, "Where is the 24 represented in the model? Where is the 6 represented in the model? Where would the unknown be represented in the model?" Using the language of rows and columns that was started in Topic 1 can help to make the connection between multiplication and division explicit. Using the array model for example, if the total number of counters and the number in each row are known, students can use that information to create an array and discover the number of counters in a column. Facilitate discussions helping students draw connections between the models, multiplication and division equations, and the inverse relationship between multiplication and division.

As a reminder from Topic 1, there are 2 different types of division problems:

Partitive (dealing or fair sharing): Number of groups is known; the size of each group is unknown

Measurement (chunking): Size of the group is known; the number of groups is unknown

Focus Math Practice 1: Make sense of problems and persevere

Focus on opportunities for students to develop Mathematical Practice 1 behaviors throughout the entire topic, as this is the focus of the Math Practices and Problem Solving lesson 4-9. Reference the Teacher's Edition (TE, pp. F21 - F21A) and the Nevada Academic Content Standards for Mathematical Practice (2010, p. 6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
dividend divisor fact family quotient	*even* *odd* *multiple* *factors* *division* *multiplication*

Additional terminology that students may need support with: *related fact, inverse relationship, opposite*

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students able to use the relationship between multiplication and division to find unknown facts?"

Lesson	Evidence	Look for
4-1	***Quick Check*** (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). • students understand that multiplication and division are inverse operations • students use the inverse operation to determine fact families Printable version available under "Teacher Resources".
4-7	***Solve & Share*** (student work samples)	Focus CTC around the big idea: • students understand that multiplication and division are inverse operations • students use the inverse operations and known facts to determine fact families and their equations

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 229-234	Use *Scoring Guide* TE pp. 229-234A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 4-1: Relate Multiplication to Division		
3.OA.B.6 **3.OA.A.3** **MP.1** MP.2 MP.4 MP.5 MP.7 MP.8	**Access Prior Learning:** In the previous topics 1-3, students learned the meaning of multiplication and multiplication facts as well as strategies for solving unknown multiplication facts. **Beginning of the Big Idea:** Students are *beginning* to understand that multiplication and division have an inverse relationship and the use of multiplication facts can help them divide numbers.	**Topic Opener:** Introduce the *Topic Essential Question*, "How can unknown division facts be found using known multiplication facts?" (TE, p. 165). Reviewing the anchor chart from Topic 3 and adding new ideas from this topic will help students to see the development of concepts and make connections. You might also consider having students complete the *Review What You Know* prior to beginning instruction on Topic 4 so that you can respond to student instructional needs using the *Item Analysis for Diagnosis and Intervention* (TE, p. 166-168). Consider introducing vocabulary as they encounter them in the lessons rather than introducing all terms at the beginning of the lesson. **Solve & Share:** During the *Solve & Share*, consider asking students, "What do we know in the model (e.g. number of rows and columns)?" "What are we solving for (total number in the array)?" "What operations could we use?" If possible, consider having a student that has noticed the multiplication and division equations use the same numbers share their solution and reasoning. Students might identify that this shows they are related facts or fact families. The *Visual Learning Animation* will explain the idea of fact family's and the inverse relationships they show. During the whole group discussion of student solution strategies and reasoning is a good time to introduce vocabulary. Connect students' informal language to formal mathematical terms. **Visual Learning:** Consider pausing the video after it asks, "How does this array show multiplication?" so that you can discuss this whole group. Depending upon the ideas shared during your classroom discussion of the Solve & Share you might also consider pausing the video after it asks, "How does this array show division?" The *Visual Learning Animation* will ask students, "What is the same about all of the equations?" and, "What is different?" after displaying a fact family. You might consider asking the same questions when doing the *Try It!* to begin to develop readiness for ideas regarding fact families and the inverse relationships they model. **-continues next page-**

		Independent Practice/Math Practices and Problem Solving: Consider assigning item 18 *Vocabulary*. This problem provides a formative assessment opportunity to check for students' understanding of fact families. Students will often think that any 3 numbers can be put together to make a fact family. Lesson 4-2 will revisit ideas involved with fact families should students demonstrate that they are still struggling. **Assess & Differentiate:** If time permits, teach students how to play *Teamwork* (TE, p. 173A). Before assigning any students the *Advanced* level activity consider asking students to play the *On-Level* with the modification that students are to create the array and show the related division fact. All students should have the opportunity to play this game. Child-watch to identify students who need additional support and pull them into a small group to do the Intervention Activity (TE, p. 173A).
Lesson 4-2: Use Multiplication to Divide with 2, 3, 4, and 5		
3.OA.B.6 **3.OA.A.3** **MP.1** MP.2 MP.3 MP.4 MP.6 MP.7	**Access Prior Learning:** In lesson 4-1 students began to develop understanding of the inverse relationship between multiplication and division and the resulting fact families. **Beginning of the Big Idea:** Students *begin* to understand that the inverse relationship between multiplication and division can be used to solve for division with a divisor of 2 through 5. Students are also *beginning* to develop the understanding that every multiplication fact has a related division fact because of their inverse relationship.	**Solve & Share:** Since students are just beginning to develop an understanding that we can use multiplication to solve for division, consider asking what strategies and tools they might use to solve this to ensure that all students have an entry point to this problem. **Visual Learning:** Consider pausing the *Visual Learning Animation* after they ask, "Why are you able to use multiplication to help you divide?" **(**00:21) to get student responses. This will provide you with formative assessment data about whether students are understanding the inverse relationship between these two operations. The *Visual Learning Animation* asks the question, "What is the division sentence?" for the first problem while showing the division sentence. You may wish to pause the video after they display the multiplication sentence and then ask your students, "What is the division sentence?" The *Visual Learning Animation* only provides one pause. It may be beneficial for your students to pause the video after they introduce each of the problems so they can have more opportunity to reason with division as an unknown factor problem while receiving immediate feedback through the video. Also consider pausing the video after they introduce Dee's sticker problem (01:33) because the problems up to this point have been Partitive (fair share) division and this is a Measurement (chunking) problem. These division types have different entry level strategies. Partitive problems allow students to use dealing into groups (one at a time or in small quantities) to fair share while measurement division problems allow students to use repeated subtraction. **Independent Practice/Math Practices and Problem Solving:** *Quick Check* item 28 *Common Core Assessment* is Measurement division (chunking) problem. Students have mostly worked with Partitive division (fair share) types so far. See the **Instructional Note** for more information regarding Measurement division. **Assess and Differentiate:** If time permits, you may consider having students play the *Teamwork* game from lesson 4-1 (TE, p. 173A). It is recommended that all students have the opportunity to play the modified version of the *Teamwork* game from lesson 4-1 before playing the *Teamwork* game provided in this lesson (TE, p. 179A). Child-watch to identify students who need additional support and pull them into a small group to do the Intervention Activity (TE, p.179A). Consider utilizing the following question formats during practice: **Example Stem 1:** Which equation has the same unknown value as 8 ÷ 2 = □? A. 8 × □ = 2 B. 2 × □ = 8 C. □ ÷ 2 = 8 D. □ ÷ 8 = 2

Lesson 4-3: Use Multiplication to Divide with 6 and 7

3.OA.B.6
3.OA.A.3

MP.1
MP.2
MP.4

Access Prior Learning:
Students have developed the understanding that inverse relationship between multiplication and division means we can use multiplication to solve for division facts. In previous lessons in this topic, they have used this understanding to solve for division facts that have a divisor of 2 through 5. In Topic 3, Grade 3 students developed strategies for solving multiplication facts with 6 and 7 as a factor.

Developing the Big Idea:
Students *begin* to understand using known multiplication facts to solve for a corresponding division fact by dividing by 6 or 7.

Solve & Share:
To assess students' readiness to use known multiplication facts with 6s or 7s as factors to solve corresponding division facts, consider asking students, "How would you explain how to solve 7 x 3 to a friend that didn't know how?" This question can activate prior learning that students will be extending to new learning. By making this question a journal response that you collect, it can also provide meaningful formative assessment data on what phase students are working in to solve for 7's facts. Students that use repeated addition or skip counting are still in Phase 1, students that used a derived fact (e.g. (2 x 7) + 7) are in Phase 2, while students that just know it and would tell them it's 21 are in Phase 3. While Phase 3 is not expected at this time, all students progress at different times and speeds.

Look Back:
Assigning the *Look Back!* will be beneficial to students still struggling to understand the inverse relationship between multiplication and division.

Visual Learning:
Consider pausing the animation at 32 seconds after it asks, "What operation should be used to solve this problem?" Accept both multiplication and division solutions with justifications.

Students that respond with multiplication should also identify the inverse relationship with division. This is an opportunity to reinforce that multiplication situations *are joining equal sized groups to find a total amount*, division situations *are separating a total amount into equal sized groups*, and to clarify any misconceptions about the two operations.

Convince Me:
It may be beneficial to assign the *Convince Me!* to continue to support conceptual development of the inverse relationship between multiplication and division.

Assess and Differentiate:
If time permits, you may consider replacing the *Problem-Solving Reading Mat* with the game *Teamwork* from lesson 4-1 (TE, p. 173A). It is recommended that all students have the opportunity to play the modified version of the *Teamwork* game from lesson 4-1. Additional game options include the game *Teamwork* from lesson 4-2 (TE, p. 179A), or the Fluency Practice Activity (TE, p. 223).

Child-watch to identify students who need additional support and pull them into a small group to do the Intervention Activity (TE, p. 185A).

Consider utilizing the following question formats during practice:

Example Stem 3: Which equation has the same unknown value as 48 ÷ 6 = □?

A. 48 × □ = 6

B. 6 × □ = 48

C. □ ÷ 6 = 48

D. □ ÷ 48 = 6

Lesson 4-4: Use Multiplication to Divide with 8 and 9

3.OA.B.6
3.OA.A.3

MP.2
MP.3
MP.4
MP.7

Access Prior Learning:
Students have developed the understanding of the inverse relationship between multiplication and division. In previous lessons in this topic, they have used this understanding to solve for division facts that have a divisor of 2 through 7.

Developing the Big Idea:
Students further *develop* their understanding of using known multiplication facts to solve for a corresponding division fact by dividing by 8 or 9.

Solve & Share:
The *Visual Learning Animation* uses a bar diagram to represent the division problem. If possible, ask a student that uses a bar diagram to accurately model the situation to share their solution strategy and reasoning.

Independent Practice/Math Practices and Problem Solving:
Consider assigning item 21 as it reviews place-value concepts.

Assess and Differentiate:
If time permits, teach students that have already had the opportunity to play the modified version of *Teamwork* from lesson 4-1 (TE, p. 173A) to play *Display the Digits* (TE, p. 191A).

Child-watch to identify students who need additional support and pull them into a small group to do the Intervention Activity (TE, p. 191A).

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		Consider utilizing the following question formats during practice: **Example Stem 2:** Which equation has the same unknown value as 27 ÷ 3 = □? A. 27 × □ = 3 B. □ = 3 × 27 C. □ × 3 = 27 D. 3 × 27 = □
Lesson 4-5: Multiplication Patterns: Even and Odd Numbers		
3.OA.D.9 **3.OA.A.3** **MP.1** MP.2 MP.3 MP.4 MP.7 MP.8	**Access Prior Learning:** In Grade 2, students learned about patterns with even and odd numbers. **Developing the Big Idea:** Students further *develop* their understanding of patterns for even and odd numbers by generalizing that all even numbers are multiples of 2. Students will also continue to *develop* their understanding of the inverse relationship between multiplication and division.	**Solve & Share:** While this lesson's focus is on finding multiplication patterns for even and odd factors, the *Solve & Share* provides the opportunity to address a common division misconception that numbers can only be divided if they can be separated into equal groups. Ask students how they figured out that 15 would not work. Watch for students that say they could not divide because 2 can't go into 15. In these cases, consider asking students to make groups of 2 from 15 counters to show that we can divide, but sometimes we will have pieces that don't fit into an equal sized group. Moving beyond this explanation goes into 4th grade standards with remainders. Also, watch for students that are able to identify that cars, boats, and books can be packaged and ask how they figured it out. Students that have an understanding of even and odd numbers will be able to explain that these numbers can be paired up perfectly with nothing left over. To connect to today's lesson, ask which factor means that we have groups of pairs (or doubles) (e.g. 2)? Students that give the response that they knew because the numbers end with 6 and 8 and those are even numbers may be working with a memorized rule as opposed to understanding. For these students, consider providing them with counters and asking them to prove their reasoning with a direct model using counters. **Independent Practice/Math Practices and Problem Solving:** If students still seem to be struggling, consider assigning item 9 *Critique Reasoning* as partner or group work and discussing whole group. **Assess and Differentiate:** If time permits, you may consider replacing the *Math and Science Activity* with the game *Teamwork* from lesson 4-1 (TE, p. 173A). It is recommended that all students have the opportunity to play the modified version of the *Teamwork* game from lesson 4-1. Additional game options include the game *Teamwork* from lesson 4-2 (TE, p. 179A), *Display the Digits* (TE p. 191A), or the *Fluency Practice Activity* (TE, p. 223). Child-watch to identify students who need additional support and pull them into a small group to do the Intervention Activity (TE, p. 191A).
Lesson 4-6: Division Involving 0 and 1		
3.OA.B.5 **3.OA.B.6** **3.OA.A.3** MP.2 MP.3 MP.6 MP.7	**Access Prior Learning:** In Topic 2, Grade 3 students developed an understanding of the Zero Property and Identity (One) Property of Multiplication. Students have also been *developing* an understanding of the inverse relationship between multiplication and division. **Developing the Big Idea:** Students further *develop* their understanding of the inverse relationship between multiplication and division to explain the division properties and patterns for division with 0 and 1.	**Instructional Note:** Students use their understanding of the Zero Property of Multiplication to explain why we cannot have 0 as a divisor. Given the problem 0 ÷ 8, which means if we start with 0 and divide it into 8 equal groups you will have 0 in each group. The *Visual Learning Bridge* (TE, p.200, Box D) explains why 0 cannot be used as a divisor. **Solve & Share:** To assess students' readiness to apply the Zero Property and the Identity Property of Multiplication to division ask students to provide an example of each property. As students share their strategies and reasoning for the *Solve & Share*, consider creating a class poster of student conjectures for division with 0 and 1 (these conjectures are stated in *Transition to the Visual Learning Bridge* (TE, p. 199). Use the poster to confirm, correct or clarify ideas presented in the *Visual Learning Animation*. The *Solve & Share* does not provide a prompt for dividing by 0. Consider asking students why there isn't a problem where 5 is divided by 0 and using the inverse operation reasoning to explain the last conjecture stated, "0 cannot be a divisor" (TE, p. 199). **Look Back:** Consider assigning the *Look Back!* as it asks students to use multiplication to justify the conjectures for division with 0. **-continues next page-**

		Assess and Differentiate: If time permits, teach students that have already had the opportunity to play the modified version of *Teamwork* from lesson 4-1 (TE, p. 173A) to play *Think Together* (TE, p. 203A). Child-watch to identify students who need additional support and pull them into a small group to do the Intervention Activity (TE, p.203A).
Lesson 4-7: Practice Multiplication and Division Facts		
3.OA.B.6 **3.OA.A.3** **3.OA.A.4** **MP.1** MP.2 MP.3 MP.4	**Access Prior Learning:** In previous lessons in Topic 4, students have used the inverse relationship between multiplication and division to solve division problems with the related multiplication fact. **Developing the Big Idea:** In this lesson, students are *developing* their understanding of the inverse relationship between multiplication and division and using a related multiplication fact to divide. Students are also *developing* understanding of using patterns and known facts to find unknown multiplication facts.	**Solve & Share:** When asking students, "What do you need to find (TE, p. 205)?" make sure that students realize they first need to find the number of people in each tour group and then how much 1 tour group will pay in entrance fees. Consider using Teaching Tool 1 **Problem Solving Record Sheet** (found in the back of the ***Teacher Resource Volume 2*** book) to help students organize their thinking as they work through this 2-step problem. **Independent Practice/Math Practices and Problem Solving:** Items 14-16 show division as the image to the right. Students have not seen division written this way and do not need to solve in this form, but it is good to expose them to conventions for division. Consider discussing this as another way to write a division problem.  **Assess & Differentiate:** If time permits, teach students that have already had the opportunity to play the modified version of *Teamwork* from lesson 4-1 (TE, p. 173A) to play *Tic Tac Toe* (TE, p. 209A). Child-watch to identify students who need additional support and pull them into a small group to do the Intervention Activity (TE, p.209A).
Lesson 4-8: Solve Multiplication and Division Equations		
3.OA.A.4 **3.OA.A.3** **MP.1** MP.2 MP.3 MP.6 MP.8	**Access Prior Learning:** In previous lessons, students have been developing reasoning strategies for solving multiplication and division facts. **Developing the Big Idea:** In this lesson, students further *develop* their understanding of strategies for solving multiplication and division facts to solve for an unknown value in an equation.	**Solve & Share:** To assess student readiness, consider asking students what the equal sign means prior to introducing the *Solve & Share*. Clarify misconceptions that the equal sign is a symbol for writing where the answer goes by informing students that the equal sign communicates a relationship of equivalence or that what is on one side of the equal sign is the same as what's on the other side. Consider also writing the equation 10 = 7 + 3 on the board and asking if this equation is correct and clarify ideas as needed. Finally consider writing 6 + 4 = 7 + 3 on the board and again ask if it is correct and clarify as needed. **Visual Learning:** Consider pausing the *Visual Learning* Animation after they ask, "What makes these equations different from other equations you have seen before?" and discuss as a whole group. If these are similar to any students' solutions to the *Solve & Share* consider asking, "How are these similar to _____'s equations that she/he used to solve the Solve & Share?" **Independent Practice/Math Practices and Problem Solving:** For item 17 *Common Core Assessment* accept responses where students write 18 ÷ 6 = ? for Part A. **Assess & Differentiate:** If time permits, you may consider replacing the *Math and Science Activity* (TE,p. 215A) with the modified version of "Teamwork" from lesson 4-1 (TE, p. 173A) as it is recommended that all students have the opportunity to play this game. Additional game options include the game *Teamwork* from lesson 4-2 (TE, p. 179A), *Display the Digits* (TE, p. 191A), *Tic Tac Toe* (TE, p. 209A), or the *Fluency Practice Activity* (TE, p. 223). Child-watch to identify students who need additional support and place them into a small group to do the Intervention Activity (TE, p.215A). Consider utilizing the following question formats during practice: **Example Stem 4:** What unknown number makes the equation true? 5 × 8 = 10 x 8 ÷ □ Enter your answer in the response box.

Lesson 4-9: Math Practices and Problem Solving: Make Sense and Persevere		
3.OA.A.3 **3.OA.D.8** **MP.1** MP.2 MP.3 MP.6 MP.8	**Access Prior Learning**: In previous lessons, students have solved 2-step questions that involved addition, subtraction, and multiplication by thinking of the hidden question. **Developing the Big Idea:** Students are further *developing* strategies for making sense of problems and persevering when they get stuck.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 1. Refer to the *Math Practices and Problem Solving Handbook* (TE, pp. F21-F21A, p. F29) for suggestions on how to develop, connect and assess this Math Practice. Also reference the handbook in the Student Edition (SE, p. F21). **Solve & Share:** Consider reintroducing MP. 1 Thinking Habits (SE, p. F21) before introducing the *Solve & Share*. Also consider using the time where students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.4 that are listed in the *Math Practices and Problem Solving Handbook* (p. F21A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice. Watch for students that try to solve the problem by using keyword strategies rather than reasoning with the context. Watch the *Listen and Look For* video prior to the lesson. Use the video to identify examples of how students might approach this problem and ways to redirect. **Convince Me:** Consider assigning the *Convince Me!* as it offers another opportunity to work with MP.1 and assess for behaviors attributed to this math practice. It also offers an opportunity to formatively assess students' understanding of the inverse relationship between multiplication division and how mathematicians can use inverse operations to check work. **Assess & Differentiate:** If time permits, you may consider replacing the *Problem Solving Reading Mat* (TE, p. 221A) with the modified version of *Teamwork* from lesson 4-1 (TE, p. 173A) as it is recommended that all students have the opportunity to play this game. Additional game options include the game *Teamwork* from lesson 4-2 (TE, p. 179A), *Display the Digits* (TE, p. 191A), *Tic Tac Toe* (TE, p. 209A), or the *Fluency Practice Activity* (TE, p. 223). Child-watch to identify students who need additional support and pull them into a small group to do the Intervention Activity (TE, p.221A).

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Kling, G. & Bay-Williams, J. (2015). Three steps to master multiplication facts. *Teaching Children Mathematics,* 21(9). 548-559.

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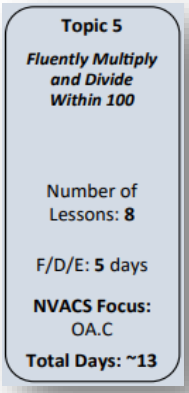
Grade 3 Topic 5: Fluently Multiply and Divide within 100



[3rd Grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: Operations and Algebraic Thinking (pp. 22-28)

Prior to instruction, view the Topic 5 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background pages (pp. 235A-235F), the Topic Planner (pp.235I-235K), all 8 lessons, and the Topic Performance Assessment (pp. 295-296A).

Mathematical Background: Read Topic 5 Cluster Overview/Math Background (TE, pp. 235A-235F)	**Topic Essential Question:** What are strategies to solve multiplication and division facts? *Reference Answering the Topic Essential Question (TE, pp. 291-292) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

5-1	5-2	5-3	5-4	5-5	5-6	5-7	5-8	Assessment

5 F/D/E days used strategically throughout the topic.

Instructional note:

This topic focuses on *securing* reasoning strategies for multiplication and division within 100 by applying the strategies needed to meet the Nevada Academic Content Standards (NVACS) for standard 3.OA.C.7, "Multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g. knowing that 8 x 5 = 40, one then knows 40 ÷ 5 = 8) and properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers" (2010). It is important to note that this expectation "know from memory all products of two one-digit numbers" is not expected until "the end of Grade 3". Encourage students to find connections between facts and look for patterns. Can they derive x4 and x6 facts based on their knowledge of x5 facts? The acquisition of basic math fact fluency occurs in three phases of development:

> Phase I: Constructing meaning and counting strategies
> Phase II: Reasoning strategies
> **Phase III: Working toward quick recall**

These three phases should not be perceived as linear. Students often work simultaneously within these phases (Washoe County School District, 2007). The Operations and Algebraic Thinking, K-2 Progression Document states, "Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g. 'adding 0 yields the same number'), and knowing some answers from the use of strategies. It is important to push sensitively and encouragingly toward fluency of the designated numbers at each grade level, recognizing that fluency will be a mixture of these kinds of thinking which will differ between students" (Common Core Standards Writing Team, 2013, p. 18). In Topic 5 students are working toward fluency. It is important to note that NVACS identify four components of fluency in its definition, which states that fluency is, "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (NVACS, 2010, p. 6). Additionally,

> This is somewhat different than "instant recall" in that it does not preclude a student's ability to use a known fact to quickly derive an unknown one. Phase III is simply about a student's quick recall. This involves increasing the speed in which the student selects and applies a strategy for solving the problem. (Washoe County School District, 2007)

Topic 5 provides the opportunity for students to continue to practice and select the most appropriate reasoning strategies introduced in previous topics. The question becomes, when do we reach Phase III? The Coherence section of this topic states,

> Throughout the rest of Grade 3, students will have many opportunities to consolidate and extend their understanding of multiplication and division and to demonstrate fluency with multiplying and dividing within 100. By the end of Grade 3, students will **know from memory** all products of two 1-digit numbers (TE, p. 235D).

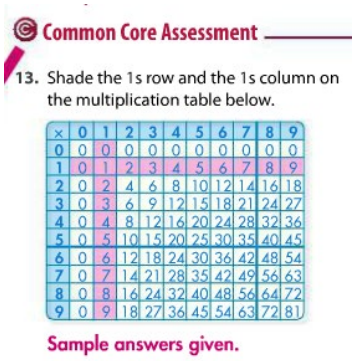
However, Phase III can only be achieved when students are provided frequent opportunities to select the most appropriate strategy through exposure to purposeful tasks and games. Prompt students to use a strategy and defend why the strategy they chose was the most appropriate. Encourage them to include the given factors and context of the problems.

While the focus of this topic is on the application of multiplication and division strategies, we must make connections to how situations can be represented with multiplication. This can be done through connecting the multiplication and division models (bar diagram, arrays, number lines) to the ideas (e.g. joining equal groups, repeated addition, known rows and columns for multiplication and separating equal groups, repeated subtraction, unknown factor for division).

Although it might be tempting to implement timed fact assessments, research indicates that timed tests tend to be harmful to students' mathematical mindsets, cause life-long math anxiety and are not correlated to developing number sense or building proficiency.

> Occurring in students from an early age, math anxiety and its effects are exacerbated over time, leading to low achievement, math avoidance, and negative experiences of math throughout life. Educators have witnessed the impact of math anxiety for decades, but only in recent years have timed math tests been shown to be one cause of the early onset of math anxiety. Indeed, researchers now know that students experience stress on timed tests that they do not experience even when working on the same math questions in untimed conditions (Boaler, 2014, p.469).

The multiplication table is used extensively in Topic 5 as a tool to connect strategies and understandings from previous topics, analyze patterns and make new connections. It is important that the multiplication table is used as a mathematical tool and not only to find products. For example, students will use the table to show the properties of multiplication as well as the inverse relationship between multiplication and division. In lesson 5-1, item 13 from **Common Core Assessment** of **Math Practices and Problem Solving** (shown to the right), illustrates the Commutative Property of Multiplication and the Identity Property of Multiplication. The multiplication table becomes a crutch for students using it to solve multiplication and division facts without attempting reasoning strategies or application of derived facts. Such procedural use will not result in students reaching Phase III, but instead will keep them dependent upon the multiplication tool to solve basic facts.


Common Core Assessment

13. Shade the 1s row and the 1s column on the multiplication table below.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Sample answers given.

Focus Math Practice 7: **Look for and make use of structure**
Focus on opportunities for students to develop Mathematical Practice 7 behaviors as this is the focus of the Math Practices and Problem Solving lesson 5-8. Reference the Teacher's Edition (TE, pp. F27 - F27A) and the Nevada Academic Content Standards for Mathematical Practice (2010, p. 8).

Looking ahead to the Topic Performance Assessment, Part B asks students to explain how they could have figured out Part A by comparing multiplication equations. In lesson 5-8 students apply MP.7 "*Look for and make use of structure*" to generate multiplication equations from contexts with comparison symbols (through reasoning) to solve a multi-step problem. Developing the thinking habits that allow students to engage in this problem type is highly beneficial.

Meaningful Fluency Practice & Assessment:
As stated above, the focus of this topic is on *securing* reasoning strategies for multiplication and division within 100 by applying an appropriate strategy and properties of operations. The version of the How Close to 100 game at the end of this document is appropriate for students that have demonstrated security with use of strategies and properties of operations to solve for facts with all factors. Games from previous topics should continue to be used based on student needs.

Phase 3: How Close to 100 (game directions and materials found at the end of this document)

> Directions: Player 1 roles 2 dice. The numbers that come up are the factors. The player then draws the array on the shared grid anywhere, so long as it does not overlap another array, and writes the equation that describes the array.
> Player 2 repeats the same process. Each player continues in turn until both players have rolled the die and cannot put any more on the grid.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
	factor *multiple* *product* *dividend* *divisor* *quotient* *Commutative (turn-around) Property of Multiplication* *Distributive (break-apart) Property of Multiplication* *Associative (order) Property of Multiplication*

Additional terminology that students may need support with: *strategy*

Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students using the properties of multiplication, tools, strategies or models to multiply whole numbers?"

Lesson	Evidence	Look for
5-1	***Quick Check*** (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). • students use their knowledge of facts and the distributive property to see patterns in factors and products on the multiplication table Printable version available under "Teacher Resources".
5-4	***Solve & Share*** (student work samples)	Focus CTC around the big idea: • students use various tools, strategies or models and properties of multiplication to find the product

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 291-296	Use *Scoring Guide* TE pp. 291-296A

Standards listed in **bold** indicate a focus of the lesson

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 5-1: Patterns for Multiplication Facts		
3.OA.D.9 **3.OA.C.7** MP.1 MP.3 **MP.7** MP.8	**Access Prior Learning:** In Topic 1, Grade 2, students used an addition table to identify patterns with addends. In Topic 3, Grade 3, students learned about the Distributive Property of Multiplication. **Securing the Big Idea:** Students are *securing* understanding of the Distributive Property of Multiplication by seeing patterns in factors and products.	**Topic Opener:** Introduce the *Topic Essential Question*, "What are strategies to solve multiplication and division facts?" (TE, p. 135). Consider making this an anchor chart in your classroom Adding new ideas each day allows students to compare and analyze new concepts and make connections throughout the topic. Ideas from the anchor charts created in Topics 3 and 4 could also be revisited during this time. Also, consider having students complete the *Review What You Know* prior to beginning instruction on Topic 5 so that you can respond to students' instructional needs using *the Item Analysis for Diagnosis and Intervention* (TE, p. 236). Introduce vocabulary as students encounter the language in the lessons rather than introducing all terms prior to instruction. **Solve & Share:** To wrap up the conversation, expose students to the formal academic vocabulary of the properties of operations by asking the question, "Can any of the properties of operations explain the patterns we found?" Assist students in connecting academic language to discovered patterns and informal observations. **Visual Learning:** To give students the opportunity to develop their abilities with MP. 7, "*Look for and make use of structure,*" consider assigning the *Convince Me!* prior to playing the *Visual Learning Animation*. The *Visual Learning Animation* explains the "double-double" strategy for facts with 4 as a factor. This strategy uses the Distributive Property of Multiplication when the 4 is decomposed into (2+2). This means that 4 x 6 can also be expressed as (2 x 6) + (2 x 6). Some students may realize that this can be solved as 2 x 2 x 6 and decide to apply the Associative Property of Multiplication to create a new problem, 2 x 12. **Assess & Differentiate:** If time permits, you may consider replacing the Math and Science Activity with games from previous topics or the Fluency Practice Activity (TE, p. 285). Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p. 113A). **CTC: *Quick Check*** (digital platform)

Lesson 5-2: Use a Multiplication Table

3.OA.C.7 MP.1 MP.2 MP.3 MP.4	**Access Prior Learning:** In Topic 4, Grade 3 students developed understanding of the inverse relationship between multiplication and division to help solve division facts. **Securing the Big Idea:** Students *secure* the understanding of multiplication as joining equal groups of objects, thinking of division as a missing factor multiplication problem, and their inverse relationship by using the multiplication table to model these connections.	**Instructional note:** As students used the addition table in 2nd grade, it may be beneficial to compare and contrast the multiplication table from the addition table at some point in the lesson. Ideas to bring out include: • Just as the addends are on the outside of the addition table, factors are on the outside of the multiplication table. • Just as the sums are on the inside of the addition table, products are on the inside of the multiplication table • The addition table can be used to solve subtraction problems and the multiplication table can be used to solve division problems. • Contrast the difference in language; addition uses addends to make sums and multiplication uses factors to make products. **Solve & Share:** Watch for students who look for any 18 in the Multiplication Chart, but don't necessarily align it with the 3 as a factor. For those that do identify 6 as the missing factor, look for understanding by asking them how they found 6 as the quotient. We want students to be able to explain that they intentionally looked for 18 in the 3's column (or row). If possible, have a student that used an understanding of the inverse multiplication/division relationship and the Multiplication Table to solve a division problem share their reasoning and explain their strategy. **Guided Practice:** For the *Guided Practice*, it is not necessary for students to do all of the items. Child-watch to determine when it's appropriate to move to the *Quick Check* items. **Independent Practice/Math Practices and Problem Solving:** Consider assigning items 24 and 25 to give students an opportunity to work with additive compare problems and reading charts. **Assess & Differentiate:** If time permits, you may consider replacing the *Problem-Solving Reading Mat* with games from previous topics or the Fluency Practice Activity (TE, p. 285). Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p.247A). Consider utilizing the following question format during practice: **Example Stem:** Enter the unknown numbers that make each equation true. 9 ÷ 3 = ☐ 28 ÷ 7 = ☐ Enter the first unknown number in the first response box.

Lesson 5-3: Find Missing Numbers in a Multiplication Table

3.OA.C.7 MP.2 MP.3 MP.4 MP.6	**Access Prior Learning:** In Topic 3, Grade 3 students developed strategies for using known facts to solve for unknown facts by using the properties of multiplication. In Topic 4, Grade 3 students developed understanding of the inverse relationship between multiplication and division to solve division facts. **Securing the Big Idea:** Students *secure* the understanding of multiplication strategies for solving for an unknown fact by using a known fact, solving for division facts by thinking of division as a missing factor multiplication problem, and their inverse	**Instructional note:** The *Solve & Share* for this lesson requires students to apply all the previous learning stated in *Access Prior Learning*. This is a very meaningful and powerful task with a high level of cognitive demand. Scaffolds may be needed to support students' engagement in productive struggle throughout the task. It is recommended that teachers engage in child watching to provide scaffolds when students demonstrate that they are in a state of struggle versus productive struggle (e.g. productive struggle has evidence of students asking questions and using reasoning to attempt the problem). **Solve & Share:** Consider asking the questions provided in *Build Understanding* (TE, p. 249) with time to think and/or share with a partner. Can students generate possible solution strategies they might attempt for finding unknowns in the multiplication chart? Share students' ideas as a whole group and post strategy suggestions. Allow student differentiated (independent, partner, or group work) time on the *Solve & Share*. After students have had time to think and collaborate, it may be beneficial to scaffold the problem for some and give the products for the 5th row down (where students need to figure out that 7 is the factor). You may consider bringing the class together to share and discuss when students are able to share some solution strategies; even if they haven't fully completed the multiplication table. ***-continues on next page-***

	relationship by using the multiplication table to model these connections.	**Assess and Differentiate:** If time permits, you may consider replacing the Math and Science Activity with games from previous topics or the Fluency Practice Activity (TE, p. 285).
Lesson 5-4: Use Strategies to Multiply		
3.OA.C.7 **3.OA.A.3** MP.1 MP.3 **MP.7**	**Access Prior Learning:** In Topic 1, Grade 3 students learned to think of multiplication as joining equal groups and skip counting as a strategy to find the total amount. In Topic 3, Grade 3 students learned to use the properties of multiplication to solve for unknown facts. **Securing the Big Idea:** Students *secure* the understanding of the various strategies introduced in Topics 2 and 3 to solve for multiplication problems.	**Instructional note** Throughout this lesson, consider frequently asking students why they chose the strategy they did and to explain their reasoning. These questions can help students to develop the 4 components of fluency as defined by NVACS (see the *Instructional Note* at the beginning of this topic for the 4 components). **Solve & Share:** To assess readiness for appropriately selecting strategies to solve multiplication problems, facilitate a class discussion using the question, "How can unknown multiplication facts be found using known facts?" Consider wrapping up your classroom discussion on student solution methods and reasoning by posing the question, "Which strategy was the most efficient and appropriate?" (Appropriate strategy use is determined by the numbers in the problem and the efficiency of the strategy.) Focus on creating a debate rather than assigning a single "best" way to solve a problem. **Independent Practice/Math Practices and Problem Solving:** Consider doing the activity described for item 31 of the *Quick Check*. It provides an opportunity to identify different multiplication strategies (The activity is described at the bottom of TE, p. 257-258). **Assess and Differentiate:** If time permits, teach students how to play "Toss and Talk" (TE, p. 259A) or a game from a previous topic that you feel students would benefit from more practice with. Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p.259A). **CTC:** ***Solve & Share*** (student work samples) Consider utilizing the following question formats during practice: **Example Stem:** Enter the unknown numbers that make each equation true. 9 × 3 = ☐ 4 × 7 = ☐ Enter the first unknown number in the first response box. Enter the second unknown number in the second response box. **Example Stem:** Select **all** expressions that equal the given product. 24 A. 6 x 4 B. 7 x 3 C. 9 x 2 D 3 x 8 E. 4 x 5
Lesson 5-5: Solve Word Problems: Multiplication and Division Facts		
3.OA.C.7 **3.OA.A.3** MP.2 MP.3 MP.4 MP.5 MP.8	**Access Prior Learning:** In Topics 1 through 4, Grade 3 students learned strategies for how to multiply and divide, as well as, the difference between a multiplication situation and a division situation. **Securing the Big Idea:** Students secure the understanding of what multiplication and division	**Solve & Share:** Consider assigning the *Look Back!* so that students have the opportunity to explain when they multiplied, when they divided, and why. **Visual Learning:** Consider assigning the *Convince Me!,* if you are noticing that students are only using a single favorite strategy to solve multiplication and division facts rather than choosing an appropriate strategy based on the numbers in the problem. ***-continues on next page-***

	are, and strategies for how to solve through real-world contexts.	**Assess & Differentiate:** If time permits, teach students how to play "Toss and Talk" (TE, p. 265A) or a game from a previous topic that you feel students would benefit from more practice with. Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p.265A).
Lesson 5-6: Write Math Stories Multiplication		
3.OA.C.7 **3.OA.A.3** MP.1 MP.2 MP.6	**Access Prior Learning:** In Topics 1 through 3, Grade 3 students learned to think of multiplication as joining equal sized groups to find a total and developed strategies. **Securing the Big Idea:** This lesson *secures* the understanding that multiplication situations involve using equal amounts of objects to find a total.	**Solve & Share:** Writing their own multiplication stories provides students with an opportunity to demonstrate their understanding of multiplicative situations and provides a formative assessment opportunity for teachers. Child watching of individual student strategies will help to identify students that are struggling to understand the difference between multiplication and addition. Watch for students that create an addition problem and support with the questions provided under *Ask Guiding Questions as Needed* (TE, p. 267). During whole class discussion of the *Solve & Share*, consider discussing Arlene's strategy (*Analyze Student Work*, TE, p. 267) and asking students if they agree or disagree that Arlene has written a multiplication problem and why. **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 11 *Number Sense*, this item provides a great opportunity for students to build their number sense through estimation. Support reasoning mathematically without solving for an exact answer. **Assess and Differentiate:** If time permits, you may consider replacing *Problem Solving Reading Mat* with either the games from previous topics, the "Teamwork" games from this topic, or the Fluency Practice Activity (TE, p. 285). Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p.271A).
Lesson 5-7: Write Math Stories Division		
3.OA.C.7 **3.OA.A.3** MP.1 MP.4 MP.5 MP.6 MP.8	**Access Prior Learning** In Topics1 & 4, Grade 3 students learned to think of division as separating an amount into equal sized groups and developed strategies for solving division problems. **Securing the Big Idea** This lesson *secures* the understanding that division situations involve separating an amount into equal sized groups and develops students' strategies for solving by asking them to write a division problem.	**Solve & Share** Writing their own division stories provides students with an opportunity to demonstrate their understanding of division situations and a formative assessment opportunity for teachers. Creating problems is often challenging, but also engaging for students. Watch for students that create the same problem type for both their division word problems. (see *Instructional note* at the beginning of Topics 1 and 4 for more details on the 2 different types of division) Students should write one problem with the number of groups as an unknown and another problem with the amount in each group as an unknown. For students that are struggling, provide support by asking the questions provided under *Ask Guiding Questions As Needed* (TE, p. 273). During the whole class discussion on student solutions and reasoning, consider using the "My Favorite No" (title is hyperlinked to a video to explain instructional strategy) with a student's work where the 2 problems are only 1 problem type. As a class, discuss whether there are 2 different division problem types present and how to revise one of the problems so that there are 2 different division problem types. **Visual Learning** Consider assigning the *Convince Me!* to help students secure their understanding that there are 2 different problem types for division. **Assess and Differentiate:** If time permits, teach students how to play "Display the Digits" (TE, p. 277A), consider asking students to write down at least 2 of the division stories they create that includes 1 of each type of division situation. Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p.277A).

Lesson 5-8: Math Practices and Problem Solving: Look for and Use Structure

MP.7
MP.1
MP.3
MP.4

3.OA.C.7

Access Prior Learning:
In Topics 2 through 4, students used the structure of numbers (decomposing numbers), the multiplication chart to identify patterns, fact families, and the properties of multiplication to solve multiplication and division problems.

Securing the Big Idea:
In this lesson, students *secure* their understanding of multiplication and division situations, and *develop* further understanding of looking for and using mathematical structures by identifying patterns and comparing equations.

Consider using an A/D/E day before moving on to lesson 5-8. See the WCSD grading/assessing documents for guidance on lesson or test items that can indicate level of achievement. Items indicated in the guides can also be used for opportunities to extend thinking.

This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 7. Refer to the *Math Practices and Problem Solving Handbook* (TE, pp. F27-F27A, F29) for suggestions on how to develop, connect, and assess this Math Practice.

Solve & Share:
Consider reintroducing MP. 7; "Look for and use structure" Thinking Habits (SE, p. F27) before introducing the *Solve & Share*. Ask students how we can use structure to use a known fact to solve for unknown facts. Consider using the time when students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.7 that are listed in the *Math Practices and Problem Solving Handbook* (TE, p. 27A). After discussing student solution methods and reasoning strategies, consider having students self-score for the behaviors associated with this math practice.

Visual Learning:
Consider assigning the *Convince Me!* as it provides an opportunity for students to apply MP.7 by explaining the use of the Zero and Identity Properties of Multiplication.

Assess & Differentiate
If time permits, teach students how to play "Clip and Cover" (TE, p. 283A) as this relates the mathematics developed in this topic to a real world context.

Child-watch to identify students who need additional support and consider the *Intervention Activity* provided (TE, p. 283A).

Consider utilizing the following question formats during practice:

Example Stem: Decide if each equation is true or false. Click True or False for each equation.

	True	False
$3 \times 6 = 18 \div 2$		
$4 \times 9 = 36 \div 4$		
$2 \times 5 = 20 \div 2$		

Example Stem: Decide if each equation is true or false. Click True or False for each equation.

	True	False
$5 \times 6 = 10 \times 3$		
$4 \times 9 = 3 \times 6$		
$8 \times 4 = 4 \times 8$		

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How Close to 100?

Mathematical Understanding:
Building fluency with factors, multiples, and recall of single-digit multiplication facts.

Grade Level: 3

Number of Players: 2

Materials Needed: How Close to 100? gameboard, 2 dice, pen or pencil

Object of the Game:
The goal is to fill up the grid to get it as full as possible.

Directions:
Player 1 roles 2 dice. The numbers that come up are the factors. The player then draws the array on the shared grid anywhere, so long as it does not overlap another array, and writes the equation that describes the array.

Player 2 repeats the same process.

Each player continues in turn until both players have rolled the die and cannot put any more on the grid.

Guiding Questions:
What are you going to try?
What did you think about to come to your answer?
Is there another way you could figure it out?
Can you think of another fact that strategy would work well for?
What equation was the hardest for you to do? Why?
What equation was the easiest for you to do? Why

Differentiation:

Each player can have their own number grid. Play moves forward to see who can get closest to 100.

Game Trajectory:
Grade 3 Fall: Players use 1 die to generate a factor and then choose the other factor from 1, 2, 5, 10 based on which will yield the array that is the most strategic.

Grade 3 Winter: Players use 1 or 2 dice depending on comfort with factors 3, 4, 6, 7, 8, and 9. Players that need more practice with one of the factors should play with 1 dice and select the difficult factor as a "fixed factor" (will be used for all arrays) and use the die to generate the other factor.

Grade 3 Spring: Players use 2 dice to generate both factors.

Clean up Checklist for Game Bag:

Copies of gameboard

2 Dice

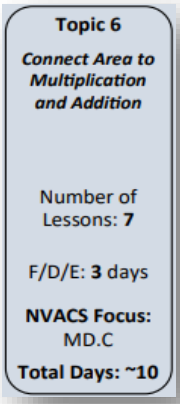
Markers, crayons, pencils, or pens

How Close to 100?

Using Known Facts

1. (___ x ___) + (___ x ___) = ___
2. (___ x ___) + (___ x ___) = ___
3. (___ x ___) + (___ x ___) = ___
4. (___ x ___) + (___ x ___) = ___
5. (___ x ___) + (___ x ___) = ___
6. (___ x ___) + (___ x ___) = ___
7. (___ x ___) + (___ x ___) = ___
8. (___ x ___) + (___ x ___) = ___
9. (___ x ___) + (___ x ___) = ___
10. (___ x ___) + (___ x ___) = ___

Grade 3 Topic 6: Connect Area to Multiplication and Addition



3rd Grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Measurement and Data (Measurement Part) (pp. 16-18)

Prior to instruction, view the Topic 6 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 297A-297F), the Topic Planner (pp.297I-297K), all 7 lessons, and the Topic Performance Assessment (pp. 353-354A).

Mathematical Background: Read Topic 6 Cluster Overview/Math Background (TE, pp. 297A-297F)	**Topic Essential Question:** How can area be measured and found? *Reference Answering the Topic Essential Question (TE, pp. 349-350) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

6-1	6-2	6-3	6-4	6-5	6-6	6-7	Assessment

3 F/D/E days used strategically throughout the topic

Instructional note:

This topic focuses on cluster heading 3.MD.C "**Geometric measurement: understand concepts of area and relate area to multiplication and division**" (2010, NVACS). This topic focuses on *beginning* to understand the measure of area. A key idea that students need to conceptualize first, is that area is an attribute of an object that can be measured and is determined by the total number of same-sized square units needed to cover a region without having gaps or overlaps. Students learn that area can be represented by an area model and described using multiplication expressions. For example, and area model with 3 rows of 4 same sized units can be described as 4+4+4 which is equivalent to 3x4.

Focus the first few lessons on understanding area and why we can't have gaps and overlaps. We have found that some children in WCSD confuse array models with area models, i.e. use array models (with circles and gaps) to try to model area (square tiles, no gaps). Provide students opportunities to solidify understanding that there are no gaps and overlaps in area models. This will be key for spatially structuring the concept of area. Reiterate explicitly throughout instruction the idea of area and what it means and how it is modeled. Consider comparing and contrasting Area and Array models, so students' area able to make this connection and clarify the purpose and intent of both models (they both can model multiplication, repeated addition etc.); yet only the area model can be used for area.

Students build on their understanding of multiplication and repeated addition to *begin* to understand area concepts. To connect this idea to prior learning from this year continue to ask students the following questions when applicable:

- What is area? How can it be measured?
- How is the area model different from the array model?
- Why can't we use an array to model area? (Gaps)
- Why is area considered a measurement? What types of things do we measure with area?
- How is area different from the linear measures we explore with data or that we explored in 2nd grade?
- How can the Distributive Property of Multiplication help us to find the total area of large objects?

Lessons 6-5, 6-6, 6-7 work with students to apply the Distributive property to compose or decompose rectilinear figures into two or more rectangles. Making this connection explicit through classroom discussion will help students to generalize these understandings and conceptualize finding the area of irregular rectilinear figures.

Note that our goal in this topic is not to formalize a formula for area, but rather to see the relationships that exist between area and the operations of multiplication and addition. This relationship is revealed through the spatial structure (MP.7) of two-dimensional shapes in the number of square units in a row and the number of rows or columns. Strongly emphasize throughout this unit that when finding area, the result is reported in square units. Students also learn that the area depends upon the size of the units used to cover the entire figure. Laying this foundation in grade 3 will provide students with the necessary prior knowledge needed to generate the formula in grade 4.

Focus Math Practice 7: **Look for and make use of structure**

Focus on opportunities for students to develop Mathematical Practice 7 behaviors throughout the entire topic, as this is the focus of the Math Practices and Problem Solving lesson 6-7. Reference the Teacher's Edition (TE, pp. F21 - F21A) and the Nevada Academic Content Standards for Mathematical Practice (2010, p. 8).

Looking ahead to the Topic Performance Assessment, students will be expected to find the area of rectilinear shapes within a larger rectilinear area, create a figure with a set area, explain why we can multiply to determine the area of a figure and explain decomposing a rectilinear area to find smaller areas or to determine the area of irregular shapes. While developing the thinking habits that allow students to engage in this problem type are highly beneficial, you may need to scaffold working with the Topic Performance Assessment.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
area unit square square unit(s)	*estimate* *addend* *array* *equal groups* *multiply* *row* *length* *inches* *feet* *centimeters* *meters* *rectangle* *square*

Additional terminology that students may need support with: *decompose, non-standard*

Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

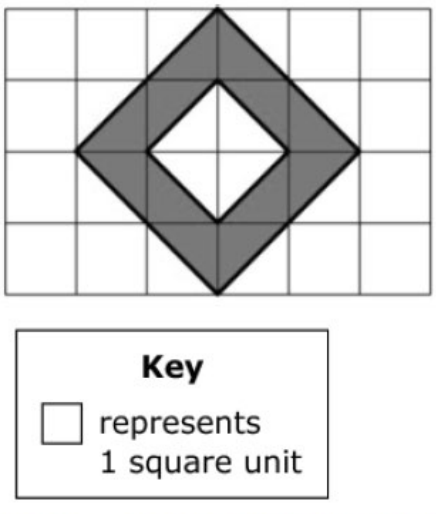
Guiding question: "Are students using their understanding of multiplication and addition to find the area of a figure?"

Lesson	Evidence	Look for
6-3	***Math Practices and Problem Solving*** (student work samples)	Focus CTC around the big idea: • students communicate the area of a shape by using standard units
6-6	***Solve & Share*** (student work samples)	Focus CTC around the big idea: • students determine the area of an irregular shape by decomposing the irregular shape into rectangles

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 349-354	Use *Scoring Guide* TE pp. 349-354A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	**Mathematical Development of the Big Idea**	**Instructional Clarifications & Considerations**
Lesson 6-1: Cover Regions		
3.MD.C.5a **3.MD.C.5b** **3.MD.C.6** MP.1 MP.2 MP.3 MP.5 MP.6	**Access Prior Learning**: At the end of second grade students covered rectangles with rows and columns of squares. **Developing the Big Idea:** Students are *beginning* to understand that the amount of space inside a shape is its area, and area can be found or estimated using unit squares.	**Topic Opener:** Introduce the Topic Essential Question, "How can area be measured and found?" (TE, p. 297). Consider making an anchor chart in your classroom. Each day new ideas are added so that students see the development and connections throughout the topic. Consider having students complete the *Review What You Know* prior to beginning instruction on topic 6 so that you can respond to students' instructional needs using the Item Analysis for Diagnosis and Intervention (TE, pp. 298-300) prior to beginning the topic. Consider introducing vocabulary as students encounter academic language in the lessons rather than introducing all terms at the beginning of the lesson. **Solve & Share:** Students will need Teaching Tool 12 found in the *Teacher's Resource Master Volume 2* to solve the problem. Watch for students that find the area by covering the shapes with the square tiles, decompose the shape, then multiply the sub-units, and add the sub-units together to find the total area. Consider having this student share their solution strategy and reasoning last as they have already connected understanding of multiplication to finding area. **Look Back:** Consider discussing as a whole group the *Look Back!* as this directly addresses a key idea in area concepts; area is a measure of two-dimensional shapes of square units needed to cover a region **without having gaps or overlaps**. Consider asking students the same question, but in this case, there are overlapping tiles. **Another Example:** Consider discussing the *Another Example!* (TE, pp. 303-304) should students still seem to be unclear about finding area using partially filled unit squares before assigning the *Quick Check* items. **Assess and Differentiate:** If time permits, you may consider replacing the *Problem Solving Reading Mat* with games from previous topics. The "Teamwork" game from lesson 3-4 (TE, p. 131A) has students practicing decomposing arrays to find the total number of objects. Revisiting this game at this time could help activate prior learning that will be generalized to finding area of shapes. Child-watch to identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p. 305A).
Lesson 6-2: Area: Nonstandard Units		
3.MD.C.6 **3.MD.C.5a** **3.MD.C.5b** MP.2 MP.3 MP.6	**Access Prior Learning**: In lesson 6-1, Grade 3 students began to understand that area can be measured by counting the number of square units in a two-dimensional figure. **Developing the Big Idea:** Students are *beginning* to understand that area can be measured using nonstandard units and to understand that the size of the area depends upon the size of the units used to cover in the rows and number of rows.	**Solve & Share:** After reading the *Solve & Share* prompt, consider discussing as a whole group the question, "What do you notice about the size of the postcard on each grid?" (TE, p. 307). It is important that students recognize they are the same size and shape. In developing understanding that the size of an area depends upon the size of the units used to cover the shape, students may form the misconception that the shapes have different areas instead of identifying the larger area measure is a result of measuring with smaller units. Watch for students that miscount the area by double counting squares. For these students suggest a method to help them keep track of squares they have already counted. **Look Back:** After the sharing of student solution strategies and reasoning, consider posing the *Look Back!* to emphasize that the measures are different because the square units are different sizes. **Convince Me:** Consider assigning the *Convince Me!* to have students apply their understanding of the impact the size of the unit square has in identifying area (e.g. students explain that the squares are alike because each has an area of 16 square units, but the squares are not the same size because the unit squares used to measure them are different sizes (TE, p. 308). ***-continues to next page-***

		Guided Practice: For item 2 in *Guided Practice,* consider asking students what they notice about the size relationship between the two unit squares. How might they use this to help them decompose the rectangles by the unit squares (e.g. for the second rectangle they could start by drawing in the larger square unit and then splitting each into fourths)? **Independent Practice/Math Practices and Problem Solving:** As a result of the difficulty many students have with the motor skills needed to draw in the smaller unit, consider only having students do the *Quick Check* items. In reviewing student work on item 5, students may need to be questioned to see if they truly do not understand the different sized square units or if motor skills are preventing them from being able to show what they know. **Assess and Differentiate:** If time permits, teach students how to play "Teamwork" (TE, p. 311A). All students should have the opportunity to play this game. Child-watch to identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p. 311A).
Lesson 6-3: Area: Standard Units		
3.MD.C.6 **3.MD.C.5a** **3.MD.C.5b** MP.2 MP.3 MP.6	**Access Prior Learning**: In previous lessons in this topic, students used non-standard unit squares to communicate the area of a two-dimensional figure. Students also learned that the size of the unit square and the size of the area depends upon the size of the units used to cover the rows and number of rows. **Developing the Big Idea:** Students *develop* understanding of communicating the area of a shape by using standard units of length, such as inches, centimeters, etc.	**Solve & Share:** Consider asking students what tool could be helpful in completing the *Solve & Share* (e.g. ruler, square tiles) and having these tools available for students. **Convince Me:** Consider assigning the *Convince Me!* Discuss whole group to reinforce the findings from lesson 6-2 that the size of the area depends upon the size of the units used to cover in the rows and number of rows. Point out that now it is determined using a standard unit of length. **Guided Practice:** Encourage students who struggle with item 1 to draw a picture to help them understand the information. Struggling students may need this strategy to solve *Quick Check* item 15. **Assess and Differentiate:** If time permits, consider replacing the *Problem Solving Reading Mat* with game "Teamwork" (TE, p. 311A). All students should have the opportunity to play this game. Child-watch to identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p. 317A). Consider utilizing the following question format during practice: **Example Stem 2:** Use this diagram to solve the problem.  Enter the area, in square units, of the shaded figure in the response box.

Lesson 6-4: Area of Squares and Rectangles

3.MD.C.7a
3.MD.C.7b

MP.1
MP.2
MP.3
MP.4
MP.8

Access Prior Learning:
In previous grades, students have learned that a square has 4 sides that measure the same length. In previous topics in Grade 3 students have used arrays to model multiplication to show the repeated addition of rows and columns to find the total.

Developing the Big Idea:
In this lesson students further *develop* their understanding of area by *beginning* to understand the relationship between area and multiplication.

Solve & Share:
Consider asking students what they know about squares to assess students' readiness for finding the area of a square. Students need to recall that a square has 4 sides that measure the same length. This is necessary so that they are able to apply the understanding to finding the area of a square where only one side's measure is given.

Watch for students that do not recognize that when we are finding the area of a square, the other side's measures will also be 6 meters. Revisit what we know about squares that can help determine the area of the shape.

Watch for students that solve using Kyoko and Shelly's work (TE, p. 319). Consider having a student whose solution method is similar to Kyoko's share first as most students in the class should be able to understand the method of drawing in the unit squares and counting each unit square. Consider having the student whose solution method is similar to Shelly's share last as this student understands how area measurements connect to multiplication. This understanding may be confirmed, clarified, or corrected during the *Visual Learning Animation*.

Visual Learning:
Consider pausing the *Visual Learning Animation* after it shares another way to find the area by counting the number of rows and multiply by the number in each row. Ask students, "Why can we multiply?" (e.g. we have equal rows of 6 so we could skip count by 6 or use repeated addition of 6.) Throughout the *Visual Learning Animation* consider asking students, where is the repeated addition in this shape, to facilitate connecting multiplication to area.

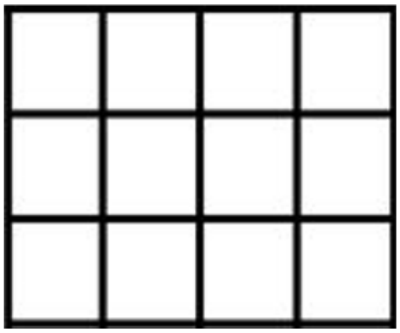
Independent Practice/Math Practices and Problem Solving:
For *Quick Check* item 7 encourage students to use what they know of multiplication and division to solve for the unknown side length. Students did this in topic 4 when they compared and contrasted what was known and unknown in a multiplication array versus a division array. Item 11 of the *Quick Check* offers distributed practice of multiplication concepts learned in lesson 2-4 *Multiply by 10* to an area context.

Assess and Differentiate:
If time permits, teach students how to play "Clip and Cover" (TE, p. 323A). Consider providing students with centimeter grid paper so that they can draw, see, and label the rectangles and areas they are creating. All students should have the opportunity to play this game.

Child-watch to identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p.323A).

Consider utilizing the following question format during practice:

Example Stem: This figure is tiled with square units.



Which expression could be used to find the area of this figure in square units?

A. 3 + 4

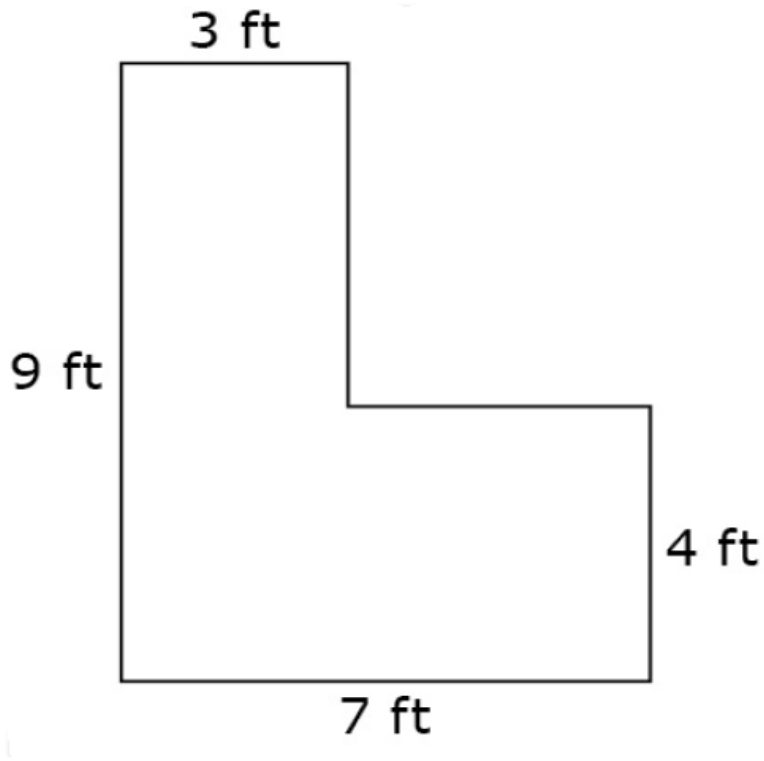
B. 3 × 4

C. 3 + 3 + 4 + 4

D. 3 × 3 × 4 × 4

Rubric: (1 point) The student chooses the correct expression (e.g., B).

Lesson 6-5: Apply Properties: Area and the Distributive Property		
3.MD.C.7c MP.1 MP.3 MP.4 **MP.7** MP.8	**Access Prior Learning**: In topic 3, Grade 3 students used the Distributive Property of Multiplication to break a large array into smaller arrays of known facts to solve for unknown multiplication problems. **Developing the Big Idea:** Students further *develop* understanding of area as the measure of unit squares inside a shape by modeling the Distributive Property of Multiplication using rectangles.	**Solve & Share:** After students have shared their solution methods and reasoning, consider asking if they could find the full area of the floor. Ask, "How does this connect to the Distributive Property of Multiplication?" Students will more easily be able to understand how to find the area of irregular shapes if in this lesson they connect using the Distributive Property of Multiplication to solving for unknown multiplication facts by breaking a larger array into smaller arrays of known facts. **Visual Learning:** To help students understand that decomposing the rectangle into smaller rectangles does not change the total area, connect topic 3 learning of breaking larger arrays into smaller arrays of known facts to find the product of unknown facts. Consider assigning the ***Convince Me!*** and having students post their solution methods for a carousel walk. Focus the walk on identifying different ways to break up the large area into smaller areas. **Assess and Differentiate:** If time permits, you may consider replacing *Math and Science Center Activity* with either the game "Teamwork" (TE, p. 311A), "Clip and Cover" (TE, p. 323A) or the *Fluency Practice Activity* (TE, p. 343).
Lesson 6-6: Apply Properties: Area of Irregular Shapes		
3.MD.C.7d MP.1 MP.2 **MP.7** MP.8	**Access Prior Learning**: In previous lessons in this topic, students learned how to find the area of rectangles building on their understanding of multiplication and the Distributive Property of Multiplication to decompose large areas into smaller areas of known multiplication facts. **Developing the Big Idea:** In this lesson, students further *develop* their understanding of area by exploring that the area of irregular shapes can be found by dividing the original shapes into rectangles, finding the area of each rectangle, and adding all of the areas.	**Solve & Share:** After introducing the *Solve & Share* consider asking students how today's *Solve & Share* is similar to what they have done in previous lessons in this topic. Pose questions to get students to share out the following ideas: • they are still finding the area of a shape • they can decompose the shape so that they are still finding the area of rectangles. For struggling students, you may want to offer geoboards, or if geoboards are not available use centimeter grid paper (Teaching Tool 13). Ask students how this tool might help them find the area (e.g. students would need to redraw the figure with each square centimeter being equivalent to a foot on the drawn figure in their book). For students that use the grid paper and count each individual square centimeter consider pairing them with a student that decomposed the shape into rectangles and solved by multiplying the sides of each decomposed rectangle and then added the areas. After students have shared their solution method and reasoning (if they haven't already explained how they knew they could multiply the sides to get the area) consider posing a question that will make the connection to multiplication explicit. **Convince Me:** After viewing the *Visual Learning Animation***,** consider having students solve the *Convince Me!* with geoboards, or if geoboards aren't available use centimeter grid paper (Teaching Tool 13). Have students share the different ways they could divide the shape. As a whole class, discuss how all the shapes still have the same area. A common misconception students will often develop is that changing the way they decompose the original shape will change the area measurement. After students have reviewed each other's solutions consider asking students what was the most efficient way to decompose the shape? **Guided Practice:** Item 4 on *Guided Practice* requires students to reason with the measures offered to determine the measures of unknown sides. Consider asking students that figured it out to share with the whole class how they figured out the measures of the unknown sides. **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 10 to formatively assess students' development of the mathematical vocabulary in this topic. **Assess and Differentiate:** If time permits, you may consider replacing the *Problem Solving Reading Mat* with either the game "Teamwork" (TE, p. 311A), "Clip and Cover" (TE, p. 323A) or the Fluency Practice Activity (TE, p. 343). Child-watch to identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p.335A). ***-continues to next page-***

		Consider utilizing the following question format during practice: **Example Stem:** This figure is made by joining two rectangles.  Enter the total area, in square feet, of the figure in the response box.
Lesson 6-7: Math Practices and Problem Solving: Look for and Use Structure		
3.MD.C.7a **3.MD.C.7b** **3.MD.C.7d** **MP.7** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6	**Access Prior Learning**: In this topic students have developed an understanding of area and how to find the area of regular and irregular shapes. **Securing the Big Idea:** In this lesson, students *secure* their understanding of area and finding the area of irregular shapes by applying MP. 7 to find the area in real-world contexts.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 7. Refer to the *Math Practices and Problem Solving Handbook* (TE, pp. 27A-27F, F29) for suggestions on how to develop, connect and assess this Math Practice. Also, reference the handbook in the Student Edition (SE, p. 27F). **Solve & Share:** Consider reintroducing MP. 7 Thinking Habits (SE, p. 27F) before introducing the *Solve & Share*. Consider using the time when students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.7 that are listed in the *Math Practices and Problem Solving Handbook* (TE, p. 27A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice. **Assess and Differentiate:** If time permits, teach students how to play "Display the Digits" (TE, p. 323A). All students should have the opportunity to play this game. Child-watch to identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p. 95A).

References

Common Core Standards Writing Team. (2012). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Measurement and Data*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

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Grade 3 Topic 7: Represent and Interpret Data

Topic 7
Represent and Interpret Data
Number of Lessons: **5**
F/D/E: **3** days
NVACS Focus: MD.B
Total Days: ~8

[3rd Grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: Measurement and Data (Data Part) (pp. 7-8)

Prior to instruction, view the Topic 7 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 355A-355F), the Topic Planner (pp. 355I-355J), all 5 lessons, and the Topic Performance Assessment (pp. 399-400A).

Mathematical Background: Read Topic 7 Cluster Overview/Math Background (TE, pp. 355A-355F)	**Topic Essential Question:** How can data be represented, interpreted and analyzed? *Reference Answering the Topic Essential Question (TE, pp. 395-396) for key elements of answers to the Essential Question.*

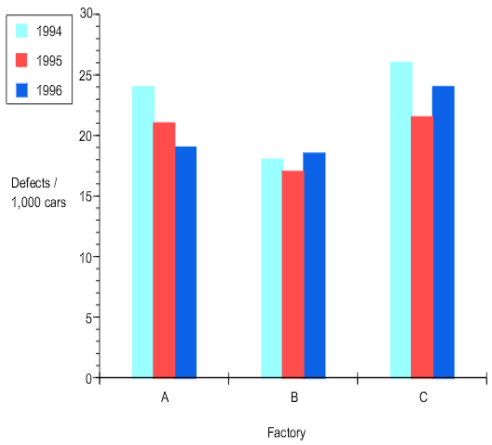
The lesson map for this topic is as follows:

7-1	7-2	7-3	7-4	7-5	Assessment

3 F/D/E days used strategically throughout the topic

Instructional Note:

In this topic, students explore and create picture graphs and bar graphs and learn to interpret categorical data. Categorical data is data that can be grouped by category or attribute. As a result, **an important misrepresentation to look for and address is having the bars touch**. In bar graphs, the bars should not be touching unless we have grouped categories of a main category. See image 1 for an example of this type of bar graph. Students do not encounter scenarios such as this in grade 3. For those interested in deepening their own content knowledge consider this resource from NC State University (https://www.ncsu.edu/labwrite/res/gh/gh-bargraph.html).



An important idea that students will be exploring is scaling. This builds on the work done this year in multiplication. Students will be using scaled graphs to interpret the data represented. Students determine the most appropriate scale for representing data shown in a frequency table that will be transferred to a picture graph or a bar graph.

The K-5, Measurement and Data (Data Part) progression document states that, "They (students) can solve one- and two-step 'how many more' and 'how many less' problems using information present in scaled bar graphs" (p.7). Students will have an opportunity to work with several different problem types, some that may prove challenging. When working with contextual problems avoid connecting specific words to a specific operation, often referred to as "keyword strategies." In the article 13 Rules that Expire, Karp, Bush, and Dougherty (2014) state:

> Using keywords often encourages students to strip numbers from the problem and use them to perform a computation outside of the problem context. Unfortunately, many keywords are common English words that can be used in many different ways…reducing the meaning of an entire problem to a simple scan for key words has inherent challenges. Keywords become particularly troublesome when students begin to explore multistep word problems because they must decide which keywords work with which component of the problem (Clement & Bernard, 2005, p, 21).

Finally, the Measurement and Data (Data Part) progression document states that students can collect their own data in the context of other content areas that can be communicated through either a picture graph or bar graph. The *Math and Science Project* for this topic does provide the opportunity for students to collect and represent data. This may be a good extension for students that are ready for this activity. It is important to note that the progression document also states that, "The standards in grades 1 through 3 do not require students to gather categorical data" (p. 7).

Focus Math Practice 6: **Attend to precision**

Focus on opportunities for students to develop Mathematical Practice 6 behaviors, as this is the focus of the Math Practices and Problem Solving lesson 7-5. Reference the Teacher's Edition (TE, pp. F26 - F26A) and the Nevada Academic Content Standards (NVACS) for Mathematical Practice (p. 7).

Looking ahead to the Topic Performance Assessment, students will be expected to solve problems from data presented in scaled bar and picture graphs that use comparative language found in the compare problem types for NVACS Math p. 88-89. Students will also

be expected to self-select from a variety of options the most appropriate scale and create a picture graph based on a provided set of data. Throughout instruction, to support students' development of selecting an appropriate scale for a graph based on data, consider frequently discussing with students the reason why the scales were chosen in the graphs. While developing the thinking habits that allow students to engage in this problem type are highly beneficial, you may need to scaffold working with the Topic Performance Assessment.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
data scaled picture graph key scaled bar graph frequency table survey	*tally* *multiplication* *equal groups* *number line* *multiples* *scale* *graph*

Additional terminology that students may need support with: *information, symbol, conclusion, analyze, record*

Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students able to represent, interpret and analyze data?"

Lesson	Evidence	Look for
7-1	***Quick Check*** (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). • students analyze and interpret scaled pictures and bar graphs. • students understand the key shows the units used. Printable version available under "Teacher Resources".
7-3	***Solve & Share*** (student work samples)	Focus CTC on the big idea: • students can make, read and analyze bar graphs using information from a table.
7-4	***Math Practices and Problem Solving*** (student work samples)	Focus CTC sense-making and problem-solving: • students can use information in a graph to solve **multi-step** problems

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 395-400	Use *Scoring Guide* TE pp. 395-400A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	**Mathematical Development of the Big Idea**	**Instructional Clarifications & Considerations**
Lesson 7-1: Read Picture Graphs and Bar Graphs		
3.MD.B.3 **3.OA.A.3** MP.2 **MP.6** MP.7 MP.8	**Access Prior Learning:** In Grade 2 students read and interpreted bar and picture graphs with single-unit scales. **Developing the Big Idea:** Students are *developing* understanding using scaled picture and bar graphs to compare data. Students develop understanding of how to interpret data in a scaled picture, using the key and that the scale for a bar graph shows the units used.	**Topic Opener:** Introduce the *Topic Essential Question*, "How can data be represented, interpreted, and analyzed?" (TE, p. 355). Consider having students complete the *Review What You Know* prior to beginning instruction on topic 3 to respond to students' instructional needs using the *Item Analysis for Diagnosis and Intervention* prior to beginning the topic (TE, p. 356-358). Consider introducing vocabulary as students encounter the academic language in the lessons rather than introducing all terms at the beginning of the lesson. ***-continues on next page-***

		Solve & Share: Consider making sure, that after you pose the question, "What do you need to use to solve this problem?" (TE, p. 359) students recognize the key and demonstrate understanding of what it means for the picture graph. Consider ensuring that the last student that shares, understands the structure of multiplication and can explain how they used multiplication to solve. **Visual Learning:** Consider pausing the *Visual Learning Animation* after the final pause when it asks, "What is another way to find how many more teams the East Falls League has than the South Falls League?" If students recognize that they can just compare the two rows and identify the difference between the two rows (e.g. there are 3 more hockey sticks in East Falls row than in South Falls row) discuss which solution method (calculating both teams amounts and subtracting or identifying the difference in rows) is more efficient. If students do not come up with the method of comparing the rows, play the animation and then discuss efficiency. After viewing and discussing the *Visual Learning Animation* consider asking students why using a scaled picture graph would be helpful (e.g. a scaled picture graph more easily represents larger sets of data). **Another Example:** Consider discussing the *Another Example* as a class as students have only reasoned with scaled picture graphs. Consider asking students how the two graphs are similar (e.g. both represent data with a scale, both are useful for comparing data, both having heading, items being compared that are identified, etc.) **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 7 to provide students with distributed practice using comparative symbols. **Assess and Differentiate:** Consider using the *Math and Science Activity* as stated in the Instructional Note above or consider replacing with games from previous topics or the Fluency Practice Activity (TE, p. 389). Child-watch to identify students who need additional support and pull them in a small group to do the *Intervention Activity* (TE, p. 389A). ***CTC: *Quick Check*** (digital platform)
Lesson 7-2: Make Picture Graphs		
3.MD.B.3 **3.OA.A.3** MP.1 MP.2 MP.3 MP.4 **MP.6**	**Access Prior Learning:** In lesson 7-1, Grade 3, students learned about scaled picture graphs. In topic 4, Grade 3, students developed understanding of "half" as division by 2. **Developing the Big Idea:** Students further *develop* their understanding of using picture graphs to represent data by finding that the key for a picture graph determines the number of pictures needed to represent the data.	**Solve & Share:** Consider posing the question from *Look Back!* while students share their solution methods and reasoning. After students have shared solution methods and reasoning consider asking students if there is another scale that could have worked given the data (e.g. 5 because all of the balls are multiples of 5). **Convince Me:** Consider assigning the *Convince Me!* to provide students with the opportunity to reason with representing half quantities on a picture graph. **Independent Practice/Math Practices and Problem Solving:** Item 4 provides students the opportunity to practice reasoning with selecting an appropriate scale based on data. Consider also assigning item 7 to ensure students have sufficient opportunity to reason with selecting a scale based on a set of data and creating a picture graph. Watch to ensure that students have included all the elements of a picture graph: • Title • Category labels • Key that includes the scale • Accurately represent the data for each category **Assess and Differentiate:** If time permits, teach students how to play "Teamwork" (TE, p. 369A). All students should have the opportunity to play this game. Child-watch to identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p. 369A). ***-continues on next page-***

Homework & Practice:
Consider solving item 8 *Common Core Assessment* in class. This item is designated as an *Advance* item; therefore, it is not appropriate to assign as independent work for all students. However, this would work as a *Solve & Share* for creating an additional lesson, if needed.

Consider saving item 2, to be completed independently in class, after lesson 7-3 as part of the formative assessment process to check students' reasoning with selecting an appropriate scale based on a set of data.

Consider utilizing the following question format during practice:

Example 1
Full Statement

Example Stem 1: Marco and Beth each read the number of books shown.

Student	Number of Books Read
Marco	12
Beth	21

Click in each row to create a picture graph that shows the number of books each student read.

Student	Number of Books Read
Marco	
Beth	

Key
represents 3 books

Rubric: (1 point) The student creates a picture or a bar graph to show the correct number for each category of data (e.g., shown below).

Student	Number of Books Read
Marco	
Beth	

Key
represents 3 books

Lesson 7-3: Make Bar Graphs

3.MD.B.3
3.OA.A.3

MP.1
MP.2
MP.3
MP.4
MP.5
MP.6

Access Prior Learning:
In lesson 7-1, Grade 3, students learned about scaled bar graphs. In topic 4, Grade 3, students developed understanding of "half" as division by 2.

Developing the Big Idea:
Students further *develop* their understanding of using bar graphs to represent data by finding that the scale determines how long each bar needs to be to represent every number in a data set.

Math Anytime:
To ensure sufficient opportunity for students to practice reasoning with selecting an appropriate scale based on data consider assigning item 4 in the *Common Core Review* (TE, p. 365A).

Solve & Share:
Watch for and correct students that draw graphs with bars for different categories touching. For more information on this common error read the Instruction Note at the beginning of this topic.

Look Back:
To support students' development of MP. 6 "*Attend to precision,"* when constructing a bar graph consider assigning and discussing the *Look Back!.*

Independent Practice/Math Practices and Problem Solving:
Consider including item 6 to ensure students have sufficient opportunity to reason with selecting a scale based on a set of data and creating a bar graph. Watch to ensure that students have included all the elements of a bar graph:

- Title
- Category labels
- Scaled quantities
- Accurately represent the data for each category

Assess and Differentiate:
If time permits, you may consider replacing the Problem-Solving Reading Mat with the game "Teamwork" (TE p. 369A) or the Fluency Practice Activity (TE p. 389).

Based upon child-watching, identify students who need additional support and pull them in a small group to do the Intervention Activity (TE p. 375A).

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Homework & Practice:
Consider saving item 4, to complete independently in class, after lesson 7-4 as a formative assessment on students' reasoning with selecting an appropriate scale based on a set of data and creating a scaled bar graph.

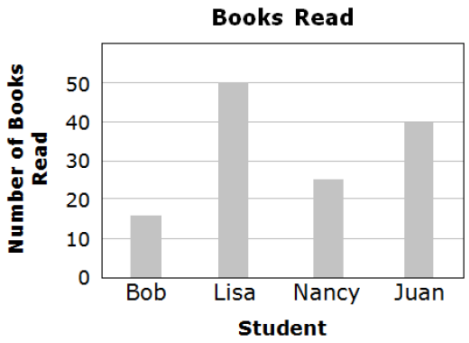
CTC:** ***Solve & Share (student work samples)

Consider utilizing the following question format during practice:

Example 2
Full Statement

Example Stem 2: Four students read the number of books shown.

Student	Number of Books Read
Bob	15
Lisa	50
Nancy	25
Juan	40



Lesson 7-4: Solve Word Problems Using Information in Graphs

3.MD.B.3
3.OA.A.3
3.OA.D.8

MP.1
MP.3
MP.6
MP.8

Access Prior Learning:
In previous grades, students have developed understanding of the operations addition and subtraction. In topics 1 through 5, Grade 3, students have developed understanding of the operations multiplication and division. In previous lessons in this topic, students developed understanding of and created scaled picture and bar graphs.

Developing the Big Idea:
In this lesson, students further *develop* understanding of scaled picture and bar graphs by making, reading, and analyzing them to solve real-world problems using the 4 operations.

Possible two-day lesson

Solve & Share:
Consider asking students what they must find (e.g., the difference of how many students prefer peanut butter more than cheese sandwiches **and** the difference of how many students prefer peanut butter more than tuna sandwiches) prior to letting students work on the problem.

Look Back:
Consider discussing the *Look Back!* as it requires that students reason with key ideas when analyzing a scaled bar graph.

Independent Practice/Math Practices and Problem Solving:
Notice that *Quick Check* items 9 and 12 flip the axis for the numbers and categories. Watch for and support students that struggle with this change. Consider discussing with the class how this change also changes the look of the scaled bar graph (e.g., the bars are horizontal now instead of vertical).

Assess and Differentiate:
If time permits, you may consider replacing the Math and Science Activity with the game "Teamwork" (TE, p. 369A) or the Fluency Practice Activity (TE, p. 389)

Child-watch to identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p. 381A).

Lesson 7-5: Math Practices and Problem Solving- Precision

3.MD.B.3
3.OA.A.3

MP.6
MP.1
MP.2
MP.4
MP.7

Access Prior Learning:
Throughout this topic, students have used precise language and symbols when analyzing scaled picture and bar graphs to solve word problems.

Securing the Big Idea:
In this lesson, students *secure* their understanding of making, reading, and analyzing scaled picture and bar graphs by focusing on accuracy (MP.6).

This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 6. Consider replacing this lesson with a task that allows students to collect and organize their own data. The task should use multiple pieces of data and provide a real-world context that allows students to see how data can be analyzed to answer questions and solve problems relevant to their own lives. Refer to the *Math Practices and Problem Solving Handbook* (TE, pp. 26A-26F, 29F) for suggestions on how to develop, connect and assess this Math Practice. Also, reference the handbook in the Student Edition (SE, p. 26F).

Solve & Share:
Consider reintroducing MP. 6, "*Attend to precision,"* (SE, p. 26F) before introducing the *Solve & Share*, you may want to restate that this includes using precise mathematical language.

Many students have a misconception that MP. 6 only refers to precise calculations. Consider using the time where students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.6 that are listed in the *Math Practices and Problem Solving Handbook* (TE, p. 26A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice.

Look Back:
Consider discussing the *Look Back!* as the reasoning discussed here can then either be confirmed, clarified, or corrected during the *Visual Learning Animation*.

Convince Me:
Consider assigning the *Convince Me!* as it offers another opportunity to work with MP.6 and assess for behaviors attributed to this math practice.

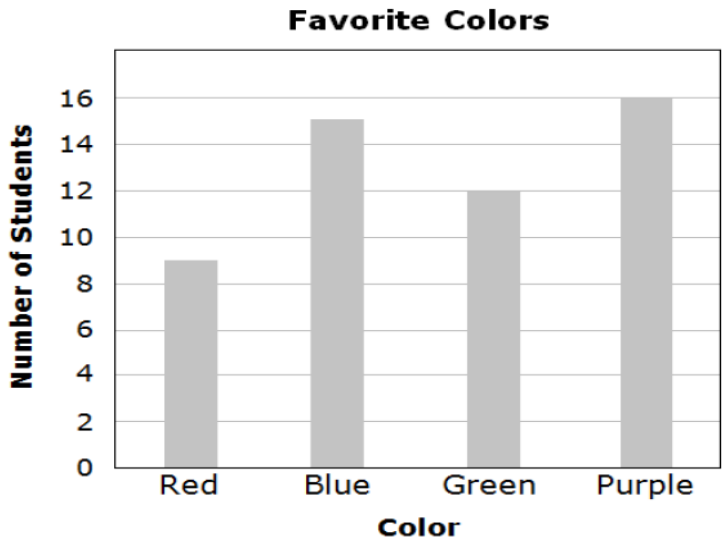
Assess and Differentiate:
If time permits, you may consider replacing the Problem-Solving Reading Mat with the game "Teamwork" (TE, p. 369A) or the Fluency Practice Activity (TE, p. 389).

Child-watch to identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p. 387A).

Consider utilizing the following question format during practice:

Example 1
Full Statement

Example Stem 1: Students voted for their favorite colors. Use the bar graph to answer the question.



How many more students voted for purple than red?

Enter your answer in the response box.

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		How many more students voted for purple and blue than green? Enter your answer in the response box. How many fewer students voted for red than purple and blue? Enter your answer in the response box. How many fewer students voted for red than purple? Enter your answer in the response box.

References

Chapin, S. H., & Johnson, A. (2006). *Math matters: Understanding the math you teach, Grades K-8*. Sausalito, CA: Math Solutions Publications.

Common Core Standards Writing Team. (2012). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Measurement and Data*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Karp, K., Bush, S., & Doughtery, B. (2014). 13 rules that expire. *Teaching Children Mathematics*, *21*(1), 18-25.

Wallace, R. (2004). Graphing resources. Retrieved from https://www.ncsu.edu/labwrite/res/gh/gh-bargraph.html

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Grade 3 Topic 11: Use Operations with Whole Numbers to Solve Problems

Topic 11
Use Operations with Whole Numbers to Solve Problems
Number of lessons: **4**
F/D/E: **4** days
NVACS Focus: OA.D
Total Days: ~8

[3rd Grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: Operations and Algebraic Thinking, K-5 (pp. 22-28)

Prior to instruction, view the Topic 11 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 571A-571F), the Topic Planner (pp.57II-571J), all 4 lessons, and the Topic Assessments (pp. 603-604A).

Mathematical Background: Read Topic 11 Cluster Overview/Math Background (TE, pp. 571A-571F)	**Topic Essential Question:** What are ways to solve two-step problems? *Reference Answering the Topic Essential Question (TE, pp. 601-602) for key elements of answers to the Essential Question.*

The lesson map for this Topic is as follows:

11-1	11-2	11-3	11-4	Assessment

4 F/D/E days used strategically throughout the topic.

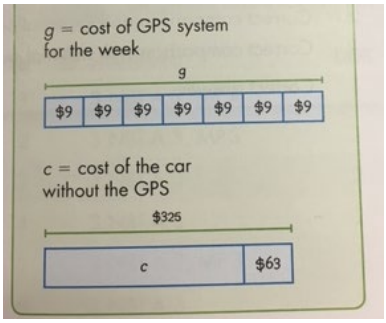
Instructional note:
In Topics 1 through 5, students developed conceptual understanding of multiplication and division. In Topics 6 & 7, students applied this understanding to area and data concepts. In Topics 8 & 9, students further developed their understanding of addition and subtraction concepts. All of this work developed operational sense.

In Topic 11 students further develop their operational sense by, "Solving problems involving the four operations…" (NVACS, 3.OA.D). Topic 11 provides students the opportunity to become more secure in this understanding as they solve two-step word problems using the four operations, identify arithmetic patterns and explain them using the properties. The focus in this topic is on algebraic thinking using, "real-world situations that can be represented using variables, operations, and equations" (TE, p. 571C). In supporting students with developing algebraic thinking, it is crucial that the algebraic language of an unknown and what the unknown represents is addressed through classroom discussion.

Third grade is the first opportunity for students to represent an unknown with a letter variable, in previous grades the unknown was represented with a box or symbol. Students should make the connection that a letter is equivalent to a question mark or an open box (Small, 2014).

> "Students need to be aware that any letter they choose is acceptable, and no one letter is preferable to another. Many teachers and students advocate using a letter that helps the student remember what the value represents. For example, in the problem "There were 24 students at 4 tables. The same number of students was at each table. How many were at each table?" students might use *s* in the equation 24 ÷ 4 = *s* to represent students in the problem. *Many* students just pick any word in the problem to suggest a letter and might use *t* from table. This, again, is not incorrect, but in the end it is critical that the students understand what their answer represents—the number of students at each table, not the number of tables" (Small, 2014, p. 31).

In this topic, students identify what the unknown is and represent it with a variable using both models and equations. To support students as they reason with two-step word problems **enVision**math**2.0** asks students, "What is the hidden question?" referring to the first step that must be answered before the stated problem can be solved. Students have worked with the idea of a "hidden question" since 2nd grade and have revisited it in previous topics.



This topic extensively uses the bar diagram to model the two-step problems, often encouraging the use of a bar diagram for each step. Encourage students to make the area of each part of the part-part-whole bar diagram representative of the quantity that part is representing. Examples in **enVision**math**2.0** model this for the students. Using representative area sizes supports students' development of magnitude of number, estimation skills (especially when the unknown is one of the parts) and being able to discern an additive situation from a multiplicative situation. As shown in the included image (TE, p. 571A), a multiplicative situation will have equal sized parts, whereas an additive situation may have different size parts.

As students' algebraic thinking develops, they will discover that some multi-step problems are solved using different operations. As a result of this idea, students will revisit the conventions for order of operations by writing a single equation that represents the multiple steps taken to solve the problem (lesson 11-3). The Operations and Algebraic Thinking progression document provides more insight into the conventions of order of operations in regards to third grade:

Understanding and using the Associative and Distributive Properties (as discussed above) requires students to know two conventions for reading an expression that has more than one operation:

1. Do the operation inside the parentheses before an operation outside the parentheses (the parentheses can be thought of as hands curved around the symbols and grouping them).

2. If a multiplication or division is next to an addition or subtraction, imagine parentheses around the multiplication or division (it is done before these operations). In Grades 3 through 5, parentheses can usually be used for such cases so that fluency with this rule can wait until Grade 6**.**

These conventions are often called the Order of Operations and can seem to be a central aspect of algebra. But actually they are just simple "rules of the road" that allow expressions involving more than one operation to be interpreted unambiguously and thus are connected with the mathematical practice of communicating precisely (MP.6). Use of parentheses is important in displaying structure and thus is connected with the mathematical practice of making use of structure (MP.7). Parentheses are important in expressing the associative and especially the distributive properties. These properties are at the heart of Grades 3 through 5, because they are used in the Level 3 multiplication and division strategies, in multi-digit and decimal multiplication and division, and in all operations with fractions (pp. 27-28).

While we are not expecting security on these conventions and it is not assessed in our Topic Assessment, lesson 11-3 focuses on bullet 2 of these conventions. Consider modeling how to write equations used in the *Solve & Share* and *Visual Learning* for all the lessons in this topic following these conventions.

Focus Math Practice 3: Construct viable arguments and critique the reasoning of others
The standard state, "They (students) make conjectures and build a logical progression of statements to explore the truth of their conjectures…. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and -if there is a flaw in and argument- explain what it is" (NVACS, 2010, p. 6-7). Behaviors associated with MP.3 are described in the Teacher's Edition (pp. F23 - F23A) and the Nevada Academic Content Standards for Mathematical Practice.

Looking ahead to the Topic Performance Assessment, item 4 Part A requires students to use their understanding of estimation and number sense to assess the reasonableness of a fictitious teacher's argument. To support students' development of the mathematical understandings needed to respond to this question consider frequently asking students if a solution is reasonable and why. For students to be successful with this assessment they will have to attend to **all** the information provided in the questions. In item 3, the word "all" makes a large impact on what students need to do to demonstrate what they know.

Therefore, throughout this topic continue to ask students if they have answered **all** of the problem and how they know. While developing the thinking habits that allow students to engage in this problem type are highly beneficial, you may need to scaffold working with the Topic Performance Assessment to help students develop these thinking habits.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
	equation *product* *quotient* *sum* *difference* *addend* *factor* *dividend* *divisor*

Additional terminology that students may need support with: *unknown, hidden question*

*Collaborative Team Conversations (CTC)

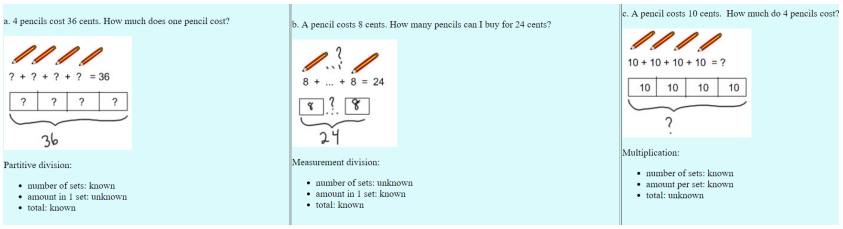
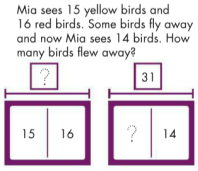
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students making sense of the problem and using appropriate operations to solve multi-step word problems?"

Lesson	Evidence	Look for
11-1	***Solve & Share*** (student work samples)	Focus CTC around the big idea: • look for students who explain the properties. • use information from table to accurately solve the problem. • all parts of multi-step problem are answered.
11-3	***Quick Check*** (digital platform) Items 1, 4, and 5	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources". • all parts of multi-step problem are answered. • make sense of problems.

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 601-604	Use *Scoring Guide* TE pp. 601-604A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 11-1: Solve 2-Step Word Problems- Addition and Subtraction		
3.OA.D.8 MP.1 MP.2 **MP.3** MP.4	**Access Prior Learning:** Throughout this year, students have had the opportunity to solve 2-step word problems with multiple operations and reasoned with the "hidden question". In Topics 8 & 9 students solved addition and subtraction problems. **Developing the Big Idea:** Students are further *developing* their understanding of 2-step word problems and operational sense by using bar diagrams and equations to model with math. Since bar diagrams represent information in the problem, students discover that bar diagrams can help them better understand what operation(s) to use.	**Topic Opener:** Introduce the *Topic Essential Question*, "What are ways to solve two-step problems?" (TE p. 571). Consider making this an anchor chart in your classroom. As new ideas are added each day, students can see the development of learning and make connections throughout the topic. You might also consider having students complete the *Review What You Know* prior to beginning instruction on Topic 11 so that you can respond to students' instructional needs using the *Item Analysis for Diagnosis and Intervention*. Consider introducing vocabulary terms as they are encountered in the lessons rather than introducing all terms at the beginning of the lesson. **Solve & Share:** To assess students' readiness, consider reviewing (students may have done this in lesson 4-1) what is known and unknown in a multiplication bar diagram versus a division bar diagram, as well as, creating bar diagrams for what is known and unknown for addition and subtraction. Multiplication/Division known/unknown bar diagram examples:  Addition/Subtraction known/unknown bar diagram examples:  The first bar diagram models addition, the second bar diagram models subtraction (TE, p. 571C). For more information on bar diagrams with the different addition/subtraction problem types go to: http://langfordmath.com/ECEMath/BasicFacts/PartWholeDiagramsText.html. ***-continues on next page-***

		If students do not offer a solution method that is the same as "Diana's Work", then consider discussing "Diana's Work" as a class. Notice that Diana uses a bar diagram to solve for both the hidden question and the final question. **Look Back:** Consider discussing the *Look Back!* prompt to help students develop habits of estimation to determine reasonableness of solutions. **Visual Learning:** Read the *Instructional Note* at the beginning of this topic for information on supporting students as they transition from using a box or symbol to represent the unknown to using a letter variable. During the *Visual Learning Animation* consider pausing during step one and step two to discuss how to solve. Also considering pausing and discussing after they pose the question, "Why is the letter *y*, instead of letter *x*, used to represent the unknown quantity in step two?" **Convince Me:** Consider discussing the *Convince Me!* prompt to help students develop habits of estimation to determine reasonableness of solutions. **Assess and Differentiate:** Child-watch to identify students who need additional support and consider the *Intervention Activity* provided. ***CTC: *Solve & Share*** (student work samples) Consider utilizing the following question format during practice: Example 1 Full Statement **Example Stem:** There are 392 students in Hall Elementary School and 503 students in Jackson Elementary School. Enter the total number of students in both schools. Enter your answer in the response box. **Rubric:** (1 point) The student enters the correct number (e.g., 935), or equivalent answer. Example 1 Full Statement **Example Stem:** There are 425 boys and 510 girls in Franklin Elementary School. How many more girls than boys are in Franklin Elementary School? Enter your answer in the response box. **Rubric:** (1 point) The student enters the correct number (e.g., 85), or equivalent answer.
Lesson 11-2: Solve 2-Step Word Problems- Multiplication and Division		
3.OA.D.8 MP.1 MP.2 **MP.3** MP.4 MP.6 MP.8	**Access Prior Learning:** Throughout this year, students have had the opportunity to solve 2-step word problems with multiple operations and reasoned with the "hidden question". In Topic 5, students solved multiplication and division problems. In the previous lesson students student discovered that bar diagrams can help them better understand what operation(s) to use. **Developing the Big Idea:** Students are further *developing* their understanding of 2-step word problems and operational sense by using bar diagrams and equations to model with math.	**Solve & Share:** Consider reviewing the differences in the known/unknown bar diagrams charts prior to introducing the *Solve & Share*. Since bar diagrams represent information in the problem, students discover that bar diagrams can help them better understand what operation(s) to use. **Look Back:** Consider discussing the *Look Back!* prompt to continue to support students' development of operational sense. **Visual Learning:** Considering pausing after being asked, "If each region didn't have the same number of teams, could you use division to find the number of teams? Why or why not?" This will allow time for students to discuss before the answer is given. **Assess and Differentiate:** If time permits, you may consider replacing the *Math and Science Activity* with games from previous topics or the Fluency Practice Activity (TE, p. 597). Child-watch to identify students who need additional support and consider the *Intervention Activity* provided.

Lesson 11-3: Solve 2-Step Word Problems- All Operations		
3.OA.D.8 MP.1 MP.2 **MP.3** MP.4 MP.7	**Access Prior Learning:** In lesson 11-1, students solved 2-step word problems with addition and subtraction, in 11-2 students solved 2-step word problems with multiplication and division, in both lessons students reasoned with the "hidden question". **Developing the Big Idea:** Students are further *developing* their understanding of 2-step word problems and operational sense with all 4 operations.	**Instructional Note:** Read the *Instructional note* at the beginning of this document regarding the conventions for order of operations. **Solve & Share:** After introducing the *Solve & Share*, consider asking the questions provided in *Build Understanding* to make sure students are able to accurately read and interpret the picture graph. **Look Back:** Consider asking the *Look Back!* prompt to continue to build students' operational sense and MP.3. **Visual Learning:** Consider pausing after the first question, to allow students to make an estimation. After viewing the *Visual Learning Animation,* consider asking students how they could write a single equation for the *Solve & Share* (e.g. 9 x 5 + 75). **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 9 *Number Sense* to practice estimation using compatible numbers. **Assess & Differentiate:** Child-watch to identify students who need additional support and consider the *Intervention Activity* provided (TE, p. 589A). ***CTC: *Quick Check*** (digital platform)
Lesson 11-4: Math Practices and Problem Solving- Critique Reasoning		
3.OA.D.8 **MP.3** MP.1 MP.2 MP.5 MP.6	**Access Prior Learning** In lesson 11-3 students continued to develop their understanding of 2-step word problems and operational sense with all 4 operations and MP.3 as they justified the operations they used. **Developing the Big Idea** In this lesson, students further *develop* their understanding of MP. 3 *"Construct viable arguments and critique the reasoning of others"* using all 4 operations to justify conjectures.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 3, *"Construct viable arguments and critique the reason of others*." Refer to the *Math Practices and Problem Solving Handbook* (TE, pp. F23-F23A, F29) for suggestions on how to develop, connect and assess this Math Practice. Also reference the handbook in the Student Edition (SE, p. F23). **Solve & Share:** Consider reintroducing MP. 3 Thinking Habits (SE, p. F23) before introducing the *Solve & Share*. Watch for students that agree with Skip's reasoning and support by asking the questions provided in *Ask Guided Questions as Needed*. Also consider using the time where students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.3 that are listed in the *Math Practices and Problem Solving Handbook* (p. F23A), and after discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice. **Look Back:** Consider facilitating a whole class discussion using the *Look Back!* question to support students' mathematical reasoning skills and place value understandings. **Visual Learning:** Consider pausing the animation to discuss Danielle's reasoning. **Convince Me:** Consider assigning the *Convince Me!* as it provides an opportunity for students to reason more with MP.3 by supporting a conjecture. **Assess & Differentiate:** If time permits, you may consider replacing the *Math and Science Activity* with games from previous topics or the *Fluency Practice Activity* (TE, p. 597). Child-watch to identify students who need additional support and consider the *Intervention Activity* provided.

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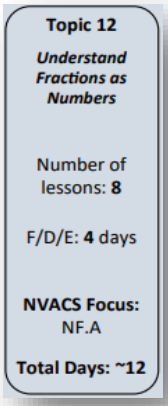
Grade 3 enVision Topic 12: Understand Fractions as Numbers



3rd Grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Number and Operations Fractions, 3-5 (pp. 3-5)

Prior to instruction, view the Topic 12 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 605A-605F), the Topic Planner (pp.605I-605K), all 8 lessons, and the Topic Assessments (pp. 667-668A).

Mathematical Background: Read Topic 12-13 Cluster Overview/Math Background (TE, pp. 605A-605F)	**Topic Essential Question:** What are different interpretations of fractions? *Reference Answering the Topic Essential Question (TE, pp. 663-664) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

12-1	12-2	12-3	12-4	12-5	12-6	12-7	12-8	Assessment

4 F/D/E days used strategically throughout the topic.

Instructional note:

Topic 12's big idea is on developing understanding of fractions as numbers (fractional number sense). Working to accurately place fractions on number line models is essential. Third grade is the first time the Numbers and Operations-Fractions domain appears in the Nevada Academic Content Standards (NVACS). However, students build fractional understanding beginning in Kindergarten. Prior knowledge that was developed in the Geometry domain (please note there are other ideas in MD that also build toward these outcomes and understandings).

Kindergarten

K.G.B.6 Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"* (NVACS, 2010, p. 12).

First

1.G.A.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrase *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares (NVACS p.16).

Second

2.G.A.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *fourths*, *half of*, *third of*, and etc., and describe the whole as two halves, three thirds, or four fourths. Recognize that equal shares of identical wholes need not have the same shape (NVACS, p. 20).

Both first and second grade work with building fractional concepts and use the language of fourths, thirds, halves, and the whole. Topic 12 begins with dividing shapes into equal regions and formally naming them as a fraction. Students name the area with unit fractions. Empson and Levi state, "The value of any fraction is determined by the multiplicative relationship between the numerator and denominator" (2010, p. 4). This relationship is seen in third grade standards with unit fractions and iterating units. Students are expected to engage in reasoning with the structure of iterations and unit fractions. They are asked to identify what the whole would look like when only shown a fractional amount of the whole. For example, students are shown $\frac{1}{3}$ of a distance on a line segment and are expected to identify which line segment represents the whole. This mathematical understanding relies on iterations, or copies of the unit fraction. When the unit fraction is known, we can then make additional copies of it until the whole has been built.

Note that students with weak spatial structuring and reasoning may struggle with this idea. Consider providing support with geoboards and linear measuring tools such as tape measures and rulers when relating to a number line. This understanding becomes critical for developing the fractional sense in fourth grade that is needed for operating on fractions. In fourth grade students will explore the idea that fractions, just like whole numbers, can be decomposed. For example, just as I can decompose 10 into 8 and 2, I can decompose $\frac{3}{4}$ into $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$.

Second grade standards are designed to develop students' spatial structures and reasoning to be able to, "Recognize that equal shares of identical wholes need not have the same shape" (NVACS, 2010, 2.G.A.3). Another important idea worth revisiting is that while our parts do have to be equal in area, they do not have to be congruent in shape (NVACS, 2010, 2.G.A.3).

Developing concept of the whole in the part-whole structure is crucial for future work in the other interpretations. Therefore, as we continue to stress the importance of what is the whole, we also discuss ideas such as, "How can we show a whole (1, $\frac{4}{4}$, $\frac{3}{3}$, etc.)",

"How do we show a fraction beyond one (whole) on the number line ($\frac{4}{3}$, $\frac{2}{1}$, etc.)?" and, "How does the fraction communicate the relationship between the amount of equal parts being discussed in the whole?"

Despite previously partitioning circles and rectangles in equal parts for halves, thirds, and quarters, doing so on a number line may be new for students; especially if they did not have sufficient opportunity with the second-grade measurement standards. As a whole-group enrichment activity it may be helpful to build a number line that stretches across the classroom starting at 0 and going to 2 that shows halves, thirds, fourths, sixths, and eighths (these are the denominators that fractions in 3rd grade are limited to per NVACS, 2010, p.25). Going beyond one whole gives students the opportunity to discuss and communicate how a fraction can be greater than one and understand that depending upon the equal number of parts needed to make the whole(s) (denominator), fractions greater than one can be named in many different ways. This will also connect to the understandings developed in 2nd grade and help link measurement to working with the number line. Providing students the opportunity to discuss the difference between $\frac{1}{2}$, $\frac{2}{1}$ and $\frac{2}{2}$ will uncover and clarify common confusions, partial understandings and misunderstandings that are often found at this grade level and better develop understanding of the role of the numerator and denominator.

Developing fractional sense offers many opportunities to confront common misconceptions. Small (2014) identifies the following common misconceptions:

- conflicts with prior knowledge about whole numbers, such as:
  - there is always a specific "next" whole number, but there is no specific next fraction.
  - 1 being the smallest number, but then finding out there are smaller numbers.
  - division makes amounts smaller, but not when dividing by proper fraction (not covered in 3rd grade).
  - 3 being more than 2, but $\frac{1}{3}$ being less than $\frac{1}{2}$; or $\frac{4}{5}$ being more than $\frac{7}{10}$ even though 7 and 10 are more than 4 and 5.
- too often using faulty perceptual arguments rather than mathematical reasoning to compare two fractions.
- viewing the numerator and denominator as separate entities (as essentially two numbers) rather than viewing the fraction as a single number.
- believing that fractions are always less than 1, perhaps because of the early emphasis on fractions as being parts of a whole, which become problematic once fractions greater than 1 are introduced.
- difficulties in placing fractions on number lines that extend past 1 (e.g., marking the point 2 when asked to place ½ on a number line that extends from 0 to 4).
- not recognizing the role the whole plays in describing a fraction.

The commonly accepted definitions for the numerator and denominator often develop the misconception of seeing each as a separate whole number. This misconception results in confusion in later grades when students are asked to operate on fractions by generalizing whole number strategies. Avoiding this misconception means we need to develop understanding of fractions as numbers the same way students developed understanding of whole numbers, by starting with counting and redefining our numerator and denominator to honor this development. Doetch (2017) explains it this way:

> "Students must learn that a fraction does not tell us anything about the size of the whole or the size of the part. A fraction tells us only about the relationship between the part and the whole".
>
> "In whole-number learning, counting helps students compare the size of numbers and later to add and subtract. This is also true with fractions. Students should come to think of counting fractional parts in much the same way as they might count with counters (bears, cubes, Unifix cubes, or other objects). When students know the parts they are counting, they can tell when they get to one whole. Students should be able to answer the following questions.
>
> 'How many thirds are in a whole?'
> 'How many fifths are in a whole?'
> 'How many twelfths are in a whole?'
>
> Counting by repeating a piece is called iterating. Understanding that $\frac{3}{4}$ can be thought of as a count of three parts called fourths. This concept becomes clear when focusing on these two ideas about fractions.
>
> - The numerator counts.
> - The denominator tells what fractional part is being counted.
>
> Another way to think of it is.
>
> - The numerator tells how many to count.
> - The denominator tells what is being counted."

Additionally, when students develop understanding of fractions as numbers by counting, we no longer need to use the language "out of" to describe a fractional amount, for example, $\frac{3}{4}$ as 3 out of 4. This language supports ratio understandings which are not explored until 6th grade.

As this is students' first formal introduction to fractions there may be need to spend more than 1 day on a lesson. The need to spend more than 1 day on a lesson should balance of being learner responsive and pacing considerations; therefore, making a lesson a 2-day lesson is a class by class decision. As a result, an additional *Solve & Share* is offered for most lessons. The *Another Look* videos could be used to fill-in for a *Visual Learning Animation*. Please note the intent is not that every lesson become a 2-day lesson, but rather, to provide the resource if it is needed.

Throughout this topic, you will notice how essential it is for our 1st and 2nd grade teachers to ensure instruction to both the Geometry and Measurement and Data Domains. Failure to instruct to *all* the standards in 1st and 2nd grade have a critical impact on forming the necessary foundations for working with fractional models and developing the necessary understandings of fractions in 3rd grade.

Focus Math Practice 1: Make sense of problems and persevere in solving them
Focus on opportunities for students to develop Mathematical Practice 1 behaviors throughout the entire topic, as this is the focus of the Math Practices and Problem Solving lesson 4-9. Reference the Teacher's Edition (pp. F21 - F21A) and the Nevada Academic Content Standards for Mathematical Practice.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
unit fraction fraction numerator denominator nearest fourth inch line plot nearest half inch	*inch* *halves* *thirds* *fourths* *yard*

Additional terminology that students may need support with: *divide (meaning to equally partition something)*

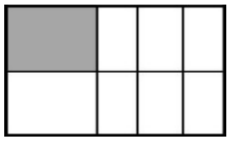
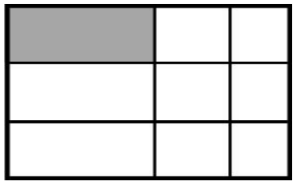
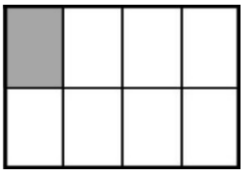
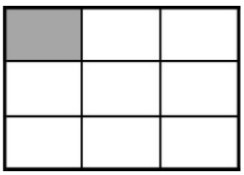
***Collaborative Team Conversations (CTC)**
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding questions: "Are students understanding the size of the whole is determined by the fractional part?"
"Are students able to accurately display data on a line plot?"

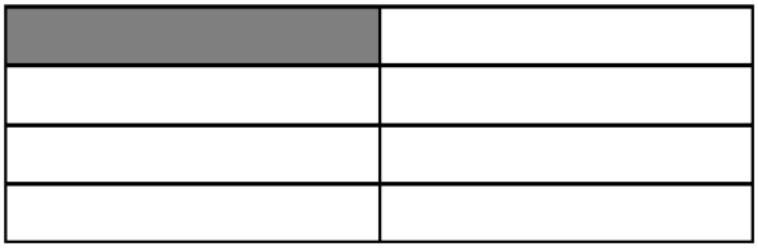
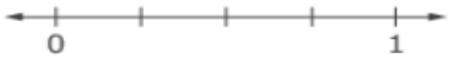
Lesson	Evidence	Look for
12-3	***Quick Check*** (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources". • understand the fraction part of the whole determines the whole.
12-7	***Solve & Share*** (student work samples)	Focus CTC around the big idea: • look for students who accurately measure sides of polygon. and display data on line plot to the nearest ½ inch. • identify most common length of polygon.

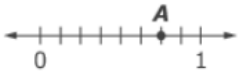
Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 663-668	Use *Scoring Guide* TE pp. 663-668A

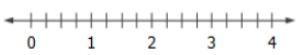
Standards listed in **bold** indicate a focus of the lesson.

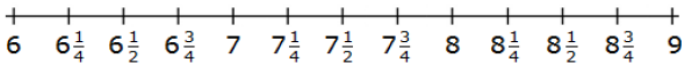
NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 12-1: Divide Regions into Equal Parts		
3.NF.A.1 **3.G.A.2** MP.1 MP.2 MP.3 MP.4 **MP.6** MP.7	**Access Prior Learning:** In Topic 15, Grade 2, students partitioned rectangles into rows and columns of same-size squares. They also partitioned circles and rectangles into halves, thirds, and fourths. **Beginning of the Big Idea:** Students are *beginning* to develop fractional sense by connecting the language of halves, thirds, and fourths to formal written form $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ while introducing $\frac{1}{6}$. Students will also *begin* developing understanding of a unit fraction.	**Instructional note:** Consider creating a class anchor chart to connect prior foundational knowledge from grades 1 (1.G.A.3) and 2 (2.G.A.3) by drawing one polygon and partitioning it into halves. Have students provide the language that each equal part is a half and label each equal part as a half of the whole polygon. Include the idea that two halves make one whole. Repeat this process with thirds and fourths (connecting the term "quarters" to fourths). **Solve & Share** Students are asked to color two different area models to show six equal parts of the whole. Students should recognize that even when colored differently, the number of equal sized parts to make the whole must be the same (denominator) to compare the number of equal sized colored parts (numerator) between the regions. **Visual Learning:** The term "unit fraction" as being single equal parts of the whole will be used in the *Visual Learning Animation*. Numerator and denominator will also be discussed. **Convince Me:** Consider assigning and discussing the *Convince Me!* as this supports the development of spatial structure in fractions. Students that are struggling with this idea may benefit from additional work with geoboards or area model representations to explore these ideas further. **Independent Practice/Math Practices and Problem Solving:** Consider assigning items 5, 6, 9, and 10 and support with geoboards or area model representations to support understanding fractions as equal parts of a whole. **Assess and Differentiate:** If time permits, consider using the *Intervention Activity* to strengthen the idea that fractions must be equal parts of the whole. Students will benefit from explaining their reasoning about how they know the parts are equal. Consider utilizing the following question format during practice: Example 1 Full Statement **Example Stem:** Figure A is divided into equal squares. One square is shaded.  **Figure A** Enter a fraction that is equal to the shaded area of Figure A in the response box. **Rubric:** (1 point) The student enters the fraction equal to the shaded portion of the shape (e.g., $\frac{1}{6}$). Example 2 Full Statement **Example Stem 2:** Which model shows $\frac{1}{4}$ of the whole figure shaded? A.  B.  C.  D.  **Rubric:** (1 point) The student selects the correct model (e.g., C).

Lesson 12-2: Fractions and Regions		
3.NF.A.1 **3.G.A.2** MP.1 MP.2 **MP.4** MP.6	**Access Prior Learning:** In Topic 15, Grade 2, students partitioned rectangles into rows and columns of same-size squares. They also partitioned circles and rectangles into halves, thirds, and fourths. **Beginning of the Big Idea:** Students are *beginning* to develop fractional sense by connecting the language of halves, thirds, and fourths to formal written form $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ while introducing $\frac{1}{6}$. Students will also *begin* understanding that a fraction represents multiple copies of a unit fraction (iterations).	**Solve & Share:** Watch for students that do not partition the rectangle into four equal size parts and support as needed. During the whole class discussion consider developing student understanding of naming parts of a whole as a fraction, as well as, fraction as number; by asking them, "How much of the garden did Pat plant flowers?" Students may likely say 3. In this case, build on their 2nd grade learning experiences by asking them, "What did we break the whole into?" At this point, support students in developing understanding of the denominator as telling us "what fractional part is being counted" and the numerator counts (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). **Look Back:** Consider discussing the *Look Back!* as a whole group to facilitate and develop students' understanding that there are 2 fractions with every representation, the one being discussed and what's not being discussed. For example, when ¾ of the whole is shaded, ¼ of the whole is unshaded. **Visual Learning:** Consider pausing and discussing after the *Visual Learning Animation* asks, "What is the whole?" Can students explain how they know what the whole is? Another pausing point to consider is, "Which parts of the fractions are the same? Why?". **Convince Me:** Consider assigning and discussing the *Convince Me!* to give students the opportunity to reason with the idea of unit fractions. **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 14 as it provides an opportunity for distributed practice of 2-step problems with multiple operations. **Assess and Differentiate:** If time permits, teach students how to play *Toss and Talk* or *Teamwork*. All students should have the opportunity to play both of these games as they provide engaging and meaningful practice of a key concept.
Lesson 12-3: Understand the Whole		
3.NF.A.3c **3.NF.A.1** MP.2 MP.3 MP.7 MP.8	**Access Prior Learning:** In previous lessons students learned that a fraction represents multiple copies (iterations) of a unit fraction. **Developing the Big Idea:** Students are beginning to understand that they can repeat copies (iterate) of a unit fraction this can determine the whole.	**Solve & Share:** To assess students' readiness consider drawing a rectangle partitioned into halves and ask students to label the unit fraction into each part prior to introducing the *Solve & Share*. For students that struggle with the idea that a unit fraction can be iterated (make additional copies of) until the whole has been built, see the *Instructional note* at the beginning of this document for ideas on how to support and consider discussing the prompts provided in *Ask Guiding Questions as Needed*. **Look Back:** Consider discussing the *Look Back!* while students share their solution methods and reasoning. This question helps students understand that there are 2 fractions with every representation, the one being discussed and what's not being discussed. This question develops an idea that will be discussed in the Visual Learning Animation, the size of the unit fraction can help to discover the size of the whole. **Visual Learning:** The *Visual Learning Animation* illustrates the part-whole relationship with the unit fraction. Students discover that the size of the whole can be determined by knowing the size of the unit fraction. This relates well to area models which make the relationship between wholes and unit fractions explicit. Consider $\frac{1}{6}$ of a personal pizza and $\frac{1}{6}$ of a family size pizza. To support students' development of this understanding, consider pausing and discussing after the following questions are posed: • Do you think your pictures for the tracks will be the same or different? Why? • What do you need in order to draw 6 lengths of 1/6? • What fraction is one whole equal to in this problem? ***-continues on next page-***

		Another Example: Consider posing the question, "How are the area model (used in the Solve & Share and Another Example) and the linear model (used in the Visual Learning) similar and different?" Help students make connections across models and recognize fraction as a number (represents a quantity) to the models. **Assess & Differentiate:** If time permits, you may consider replacing the *Math and Science Activity* with the games *Teamwork*, *Toss and Talk*, or the *Fluency Practice Activity*. Consider utilizing the following question formats during practice: Example 3 Full Statement **Example Stem 3:** The fraction model shows $\frac{1}{8}$ of the whole figure shaded.  What numerator goes in the box (☐) to make the equation true? $\frac{\square}{8} = 1$ Enter your answer in the response box. **Rubric:** (1 point) The student enters the correct value (e.g., 8). **Example Stem 2:** Use the number line to help you complete the equation.  $1 = \frac{\square}{4}$ What numerator goes in the box (☐) to make the equation true? Enter your answer in the response box. **Rubric:** (1 point) The student enters the correct value (e.g., 4).
Lesson 12-4: Number Line- Fractions Less than 1		
3.NF.A.2a **3.NF.A.2b** MP.3 MP.4 MP.6	**Access Prior Learning:** In previous lessons in this topic students learned that the denominator indicates the number of equal parts that the whole is divided into and the numerator indicates how many equal parts the fraction represents. **Developing the Big Idea:** Students further *develop* their understanding of fractions by finding that points on a number line can represent fractions. The denominator represents the number of equal parts between 0 to 1, and the numerator represents the number of parts between 0 and the point.	**Solve & Share:** Child-watch for students that have the misconception of starting with $\frac{1}{3}$ instead of $\frac{0}{3}$. In addition, child-watch for students that are confusing what the whole represents (1-mile). Consider discussing the *Look Back!* to support students' fraction reasoning. **Visual Learning:** Consider discussing the *Convince Me!* to revisit the ideas developed in lesson 12-3's Visual Learning Animation. **Assess & Differentiate:** Consider having all students do the *Intervention Activity* (TE, p. 631A) to work more with the number line model. **Possible Day 2 Solve & Share:** (Read the *Instructional note* at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using item 10 MP. 6 *Be Precise* from the *Independent Practice/Math Practices and Problem Solving*. ***-continues on next page-***

		Consider utilizing the following question formats during practice: Example 1 Full Statement **Example Stem:**  Enter the fraction located at point *A* on the number line. **Rubric:** (1 point) The student enters the fraction that is located at the point on the number line (e.g., $\frac{6}{8}$). Example 1 Full Statement **Example Stem:** Use the Add Point tool to place a point on the number line where $\frac{2}{4}$ should be located.  **Rubric:** (1 point) The student places a point at the correct location on the number line (e.g., $\frac{2}{4}$ is placed halfway between 0 and 1). Example 1 Full Statement **Example Stem 1:** Drag each fraction to the number line, as close to the exact location as possible.  $\frac{3}{8}$ $\frac{1}{8}$
Lesson 12-5: Number Line- Fractions Greater than 1		
3.NF.A.2b **3.NF.A.2a** **3.NF.A.3c** MP.3 MP.4 MP.5 MP.6 MP.8	**Access Prior Learning:** In the previous lesson students learned to use number lines to work with fractions. **Developing the Big Idea:** Students further *develop* their understanding of representing fractions on a number line by generalizing to represent fractions greater than 1 on a number line. Students further *develop* their fractional sense by developing understanding of how to represent a whole number as a fraction.	**Instructional note:** The primary purpose of this lesson is to confront the misconception that it is not possible to have fractions greater than 1. **Solve & Share:** As students generalize understanding of fractions greater than 1 to a number line, it may be helpful to relate it to fractions on a ruler. Consider discussing the *Look Back!* to support students' fraction reasoning. After students have shared their solution methods and reasoning, consider discussing the *Look Back!* to generalize understandings developed from the *Solve & Share*. NVACS standard 3.NF.A.3c calls for students to, "Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = $\frac{3}{1}$; recognize that $\frac{6}{1}$ = 6; locate $\frac{4}{4}$ and 1 at the same point of the number line diagram"* (2010). Therefore, consider holding up 2 more whole strips of paper and asking students how do represent 2 wholes as a fraction (e.g., $\frac{2}{1}$). Don't worry if students are struggling with this idea, it will be further developed in lesson 13-7. **Visual Learning:** The *Visual Learning Animation* asks, "What do the marks on the number line represent?" Consider pointing out that it is necessary to know how many equal parts there are from 0 to 1 on a number line before writing missing fractions. **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 13 *Critique Reasoning* for distributed practice of the Associative and Distributive Properties of Multiplication. **Assess & Differentiate:** If time permits, you may consider replacing the *Problem Solving Reading Mat* with the game *Teamwork* (TE, p. 619A) or *Toss and Talk* (TE, p. 613A, TE, p. 631A). **Possible Day 2 Solve & Share:** (Read the **Instructional note** at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using item 10 MP. 6 *Be Precise* from the *Independent Practice/Math Practices and Problem Solving*. ***-continues on next page-***

		Consider utilizing the following question during practice: **Example Stem 2:** Place each fraction on the number line, as close to its exact location as possible.  $\frac{2}{2}$ $\frac{1}{4}$ $\frac{4}{1}$ $\frac{2}{4}$ **Rubric:** (2 points) The student places all fractions at the correct location on the number line (e.g., $\frac{2}{2}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{4}{1}$ are placed at their approximate location). A tolerance of ± half of the unit fraction is acceptable for scoring (e.g., $\pm\frac{1}{8}$ for fourths). (1 point) The student places three out of four fractions at the correct location, within the interval of tolerance, AND places the other fraction on the correct side (less than or greater than) of the correctly placed fractions.
Lesson 12-6: Line Plots and Length		
3.MD.B.4 MP.1 MP.2	**Access Prior Learning:** In Topic 14, Grade 2, students measured to the whole inch and represented the data on a line plot. In lesson 12-4, Grade 3, students learned to represent fractions on a number line. **Developing the Big Idea:** Students further *develop* their understanding of representing fractions on a number line by generalizing understanding from lesson 12-4 and extend it to representing fractions beyond 1 on a number line.	**Instructional Note:** Standard 3.MD.B.4 states that students use rulers to collect measurement data and show the data using whole numbers, halves, or quarters. Student measure to the nearest ¼ inch before the nearest half inch to establish the ¼ marks as benchmarks for when measuring to the nearest ½ inch. **Solve & Share** Consider assessing readiness prior to introducing the *Solve & Share*, by distributing rulers and asking students what they know or notice about the inches side of a ruler. Ideas to generate in this conversation are: • When measuring objects, we start at 0. • The numbers on the ruler indicate inches. • The lines in between the inches indicate where each inch has been partitioned into halves and quarters. • Where the marks for halves and quarters marks are located. • How to read measurements that go past a whole to the nearest $\frac{1}{2}$; and $\frac{1}{4}$ inch. Child-watch for students that do not start their measurements correctly along the ruler and support as needed. Also watch for students that need support to measure to the half inch and quarter inch. After students have shared their solution methods and reasoning, consider discussing the *Look Back!* to revisit ideas from Topic 7 with bar graphs and picture graphs. **Visual Learning:** As students generalize understanding of fractions to a number line, it may be helpful to relate it to fractions on a ruler. Consider asking and discussing, "How do you know which tick mark is appropriate for reasoning with fractions of fourths?". Also discuss and revisit the ideas of scaling and precision when gathering/representing data from Topic 7. **Assess & Differentiate:** Consider doing the *Intervention Activity* (TE, p. 643A) with all students as it asks them to make a line plot using the given lengths. **Possible Day 2 Solve & Share:** (Read the *Instructional Note* at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using items 9 and 10 from the *Independent Practice/Math Practices and Problem Solving*.
Lesson 12-7: More Line Plots and Length		
3.MD.B.4 **MP.1** MP.2 MP.4 MP.5 MP.6	**Access Prior Learning:** In the previous lesson, students use what they know about number lines and fractions to understand that points plotted onto a number line create a line plot, which helps organize and interpret data to the quarter inch. **Developing the Big Idea:** In this lesson, students are further *developing* their understanding of line plots, but to the nearest half inch. Students also develop the understanding that a half-inch is two quarter inches.	**Instructional Note:** Identifying ¼ measurements on a ruler can be reinforced in lesson 12-7 as students can use ¼ and ¾ measurements to determine if the measurement is closest to a whole inch or the half inch. **Solve & Share:** Consider printing the rulers provided in Teaching Tool 19 and drawing your own polygon for students to measure as the one provided in the book does not yield consistent measurements. Continue to watch for students that do not start their measurements correctly along the ruler and support as needed. Also watch for students that need support to measure to the half inch and quarter inch. ***-continues on next page-***

		Visual Learning: After the *Visual Learning Animation* asks, "How do you know that the $3\frac{1}{2}$ - inch length occurred most often?" Consider wrapping up the discussion by pointing out that every dot on a number line represents actual data. When data changes, the line plot also needs to change. **Independent Practice/Math Practice and Problem Solving:** Consider using items 12 MP. 6 *Be Precise* and 13 *Higher Order Thinking* as students use information in a table to solve problems. Consider asking students to make a line plot of the data presented in the table. **Assess & Differentiate:** Consider having all students do the *Intervention Activity* (TE, p. 649A) as it gives students the opportunity to measure items, collect data and use the data collected to build a line plot. Consider utilizing the following question format during practice: **Example 1** Full Statement **Example Stem:** A boy measures the length of some items in his desk. This chart shows the length, in inches, of each item. **School Supply** / **Length (in)** Pencil / $7\frac{1}{4}$ Paper / $8\frac{1}{2}$ Stapler / $6\frac{3}{4}$ Paintbrush / $8\frac{1}{2}$ Marker / $6\frac{1}{2}$  6 $6\frac{1}{4}$ $6\frac{1}{2}$ $6\frac{3}{4}$ 7 $7\frac{1}{4}$ $7\frac{1}{2}$ $7\frac{3}{4}$ 8 $8\frac{1}{4}$ $8\frac{1}{2}$ $8\frac{3}{4}$ 9 **Length of School Supplies (in)** Click above the tick marks to complete the line plot that displays the data. **Rubric:** (1 point) The student correctly marks all 5 points to create the line plot.
Lesson 12-8: Math Practices and Problem Solving- Make Sense and Persevere		
3.NF.A.1 **MP.1** MP.2 MP.3 MP.6	**Access Prior Learning:** In previous lessons, students have developed an understanding of fractions as number and representing data with fractions on a line plot. **Developing the Big Idea:** Students are further *developing* their understanding of MP. 1 and fractions as numbers to analyze given information and determine what is or is not needed to solve problems in real-world contexts.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 1. Refer to the *Math Practices and Problem Solving Handbook* (TE pp. F21-F21A, F29) for suggestions on how to develop, connect and assess this Math Practice. Also reference the handbook in the Student Edition (SE, p. F21). **Solve & Share:** Consider reintroducing MP. 1 Thinking Habits (SE, p. F21) before introducing the *Solve & Share*. Also consider using the time students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.1 that are listed in the *Math Practices and Problem Solving Handbook* (TE, p. F21A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice. **Visual Learning:** Considering pausing and discussing, "How can I make sense of and solve this problem?". For all problem solving it is key that students know what the problem is asking and generate a plan for finding that information. **Assess & Differentiate:** If time permits, consider teaching students how to play the game *Teamwork* (TE, p. 655A). All students should have the opportunity to play this game as it provides engaging and meaningful practice of a key concept.

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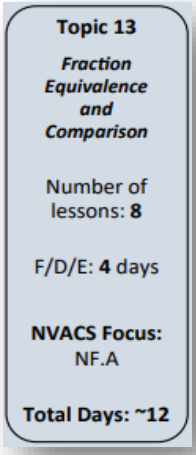
Grade 3 enVision Topic 13: Fraction Equivalence and Comparison



Big Conceptual Idea: Number and Operations Fractions, 3-5 (pp. 3-5)

Prior to instruction, view the Topic 13 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 605A-605F), the Topic Planner (pp.669A-669C), all 8 lessons, and the Topic Performance Assessment (pp. 731-732A).

Mathematical Background: Read Topic 12-13 Cluster Overview/Math Background (TE, pp. 605A-605F)	**Topic Essential Question:** What are different ways to compare fractions? *Reference Answering the Topic Essential Question (TE, pp. 727-728) for key elements of answers to the Essential Question.*

The lesson map for this Topic is as follows:

13-1	13-2	13-3	13-4	13-6	13-5	13-7	13-8	Assessment

4 F/D/E days used strategically throughout the topic.

3rd Grade Curriculum Pacing Framework: Balanced Calendar

Instructional note:

Topic 13 continues developing the idea of fractions as numbers (fractional number sense) and includes understanding that fractions can look different but have the same value (equivalence) and that fractions can be compared. The number line and area models continue to be essential high leverage models and students should use these representations to explore, demonstrate and justify their thinking throughout the topic.

The question, "What is the whole?" will be critical when working with fraction equivalence and comparison. Students must be able to identify, or make the assumption that they are working with the same size whole in order to identify equivalent fractions and make comparisons. As stated by Chapin and Johnson (2006):

> "Equivalence is one of the most important mathematical ideas for students to understand, particularly with regard to fractions. Equivalence is used when comparing fractions, ordering fractions, and adding and subtracting fractions. Equivalent fractions are fractions that represent equal value; they are numerals that name the same fractional number. When represented using a number line, equivalent fractions represent the same distance" (p.114).

The importance of the whole is key to third grade fraction standards. Van de Walle, Karp, Lovin, Bay-Williams (2014) state, "Every fraction is equal to an infinite number of other fractions" (p. 220). This idea includes seeing whole numbers as fractions as outlined in the Numbers and Operations—Fractions, 3-5 progression document (p. 4). Consider using a number line to model this understanding. Topic 13 does this throughout the topic using the double number line.

This topic explores strategies to compare nonequivalent fractions with the same whole. A key idea in comparing fractions is understanding that the larger the denominator the smaller the unit fraction, when comparing fractions with the same denominator the larger the numerator the larger the fraction. Emphasize and formalize these ideas as students discover them. Focus on providing many experiences for students to notice these patterns rather than explicitly teaching them.

Throughout this topic fraction strips and number lines are used heavily to model the ideas of fraction equivalence and comparison. Van de Walle, et. al. states, "Sometimes it is useful to do the same activity with two different representations as they offer different opportunities to learn. For example, an area model helps students visualize parts of the whole, and a linear model shows that there is always another fraction to be found between any two numbers" (2014, p. 207). It is beneficial to select students with different models to share and use these different strategies to facilitate a share and compare classroom discussion.

Consider spending more than 1 day on a lesson. The need to spend more than 1 day on a lesson should be a balance of being learner responsive and pacing considerations; therefore, making a lesson a 2-day lesson may be different from class to class. As a result, an additional *Solve & Share* for each lesson is offered. The *Another Look* videos could be used to fill-in for a *Visual Learning Animation*. *Please note the intent is not that every lesson become a 2-day lesson, but rather, to provide a resource for when it is needed*.

To support students' development of MP.3 "*Construct viable arguments and critique the reasoning of others*," consider focusing on opportunities for students to develop Mathematical Practice 3 behaviors throughout the entire topic, as this is the focus of the Math Practices and Problem Solving lesson 13-8. To support students' development of MP. 3 consider using some of the language as a

regular part of instruction. Giving students directions that include phrases such as, "construct an argument" and "justify your conjecture" may help students build understanding of what MP. 3 entails. Additional resources include the Teacher's Edition (pp. F23 - F23A) and the Nevada Academic Content Standards (NVACS) for Mathematical Practice.

Looking ahead to the Topic Performance Assessment, students should be prepared with the conventions of using a number line and double number lines. While developing the thinking habits that allow students to engage in this problem type are highly beneficial, you may need to scaffold working with the Topic Performance Assessment to help students develop these thinking habits.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
equivalent fractions (13-1)	*fraction* **numerator* **denominator* *greater than* *less than* *conjecture*

Additional terminology that students may need support with: *compare,* *Consider using the definition described in Topic 12 Curriculum Guide's *Instructional note* to avoid developing misconceptions.

Topic 13

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

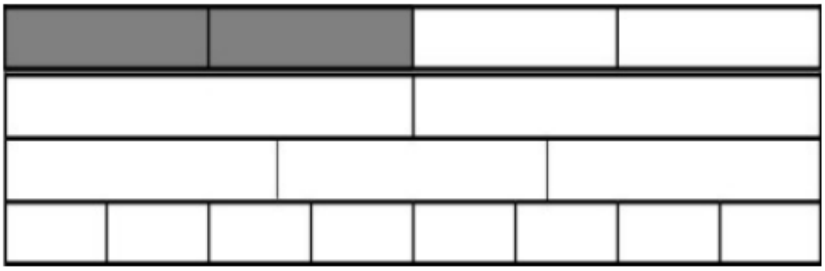
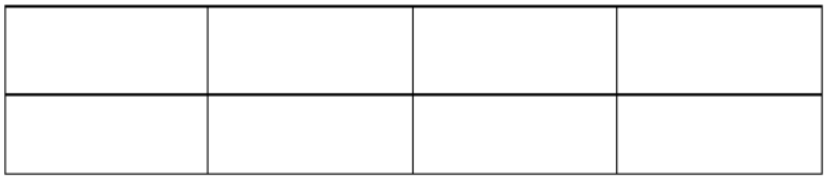
Guiding question: "Are students finding equivalent fractions and comparing fractions based on the whole using multiple strategies?" (fractions strips, number lines, models, benchmark fractions)

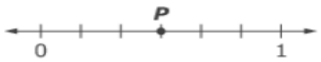
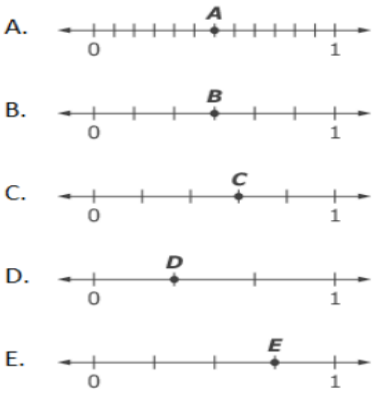
Lesson	Evidence	Look for
13-8	***Math Practices and Problem Solving*** (student work samples) Focus on items 7 and 9	Focus CTC around the big idea: • student understanding of the whole (item 7) • constructing a math argument based on comparison of fractions (item 9)
13-2	***Quick Check*** (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources". • ability to use a number line to identify equivalent fractions

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 727-732	Use *Scoring Guide* TE pp. 727-732A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 13-1: Equivalent Fractions: Use Models		
3.NF.A.3a **3.NF.A.3b** MP.2 MP.4 MP.5 MP.7	**Access Prior Learning:** In Topic 12, Grade 3, students learned that fractions name a part of a whole. **Developing the Big Idea:** Students are *beginning* to develop an understanding that the same fraction amount can be represented by an infinite set of different but equivalent fractions.	**Topic Opener:** Introduce the *Topic Essential Question*, "What are different ways to compare fractions?" (TE p. 669). Consider making this an anchor chart in your classroom that allows for the addition of new ideas so that students can see the development and connections throughout the topic. Consider having students complete the *Review What You Know* prior to beginning instruction on Topic 13 so that you can respond to students' instructional needs using the *Item Analysis for Diagnosis and Intervention* prior to beginning the topic (TE, pp. 670-672). Consider introducing vocabulary as they encounter them in the lessons rather than introducing all terms at the beginning of the lesson. ***-continues on next page-***

		Solve & Share: After introducing the Solve & Share consider asking students what tool might be helpful (e.g., fraction strips) before asking the provided question in *Build Understanding* (TE, p. 673). Watch for students that don't partition the rectangle(s) into equal parts. **Visual Learning:** Consider discussing the *Convince Me!* to make explicit a strategy for identifying equivalent fractions for $\frac{1}{2}$. The *Convince Me!* also reinforces the definition of a fraction as, "The value of any fraction is determined by the multiplicative relationship between the numerator and denominator" (Empson & Levi, 2011, p. 4). **Assess & Differentiate:** If time permits, teach students how to play *Display the Digit* (TE, p. 677A). All students should have the opportunity to play this game as it provides engaging and meaningful practice of a key concept. Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p. 677A). **Possible Day 2 Solve & Share:** (Read the Instructional note at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using items 9 and 10 MP. 4 *Model with Math* from the *Independent Practice/Math Practices and Problem Solving*. Consider utilizing the following question during practice: Example 1 Full Statement **Example Stem:** Use the fraction strip model shown to help you with this problem.  Enter a fraction equal to $\frac{2}{4}$ that has a **different** denominator in the response box. **Rubric:** (1 point) The student enters an equivalent fraction (e.g., $\frac{1}{2}$ or $\frac{4}{8}$). Example 1 Full Statement **Example Stem:** Use this model to solve the problem.  Click parts of the model to shade $\frac{2}{4}$ of the whole model. **Rubric:** (1 point) Student creates a fraction model equal to the given fraction (e.g., $\frac{2}{4}$).
Lesson 13-2: Equivalent Fractions: Use the Number Line		
3.NF.A.3a **3.NF.A.3b** **MP.3** MP.4 MP.5	**Access Prior Learning:** In Topic 12, Grade 3, students learned to represent fractions on number lines. In the previous lesson students learned about equivalent fractions. **Developing the Big Idea:** Students further *develop* their understanding of equivalent	**Solve & Share:** A common misconception when working with number lines is for students to count the hash marks instead of the sections to name the fractions. Consider discussing "Drew's Work" (TE p. 679) to confront this misconception. Consider wrapping up the class discussion of students' solutions and reasoning by discussing the *Look Back!* prompt. ***-continues on next page-***

	fractions by finding that there are limitless number of fraction names for each point on a number line. These points can be used to name equivalent fractions.	Since both the fractions strip (area) and number line (linear) models have been introduced it may be beneficial to have a class discussion on how both models represent the same mathematics through their similarities and differences. **Independent Practice/Math Practices and Problem Solving:** Consider discussing item 8 as it offers an opportunity to discuss and develop understanding of equivalent names for 1. Consider discussing item 12 *MP.3 Construct Arguments* to support students' development of MP.3, and to develop schema for the next lesson. **Assess & Differentiate:** If time permits, you may consider replacing the *Math and Science Activity* with the game *Display the Digit* (TE, p. 677A) or the *Fluency Practice Activity* (TE, p. 721). Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p. 783A). **Possible Day 2 Solve & Share:** (Read the *Instructional Note* at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using the *Convince Me!* Consider utilizing the following question format during practice: Example 1 Full Statement **Example Stem:** Use this number line to answer the question that follows.  Select **all** the number lines that show a fraction equal to the fraction shown by point *P*.  **Rubric:** (1 point) The student selects all number lines that show $\frac{1}{2}$ (e.g., A, B).
Lesson 13-3: Use Models to Compare Fractions- Same Denominator		
3.NF.A.3d MP.2 **MP.3** MP.5 MP.6 MP.8	**Access Prior Learning:** In previous lessons students used fraction strips and number lines to find equivalent fractions. **Developing the Big Idea:** Students are further *developing* fractional sense by comparing fractions with the same denominator. Students use quantitative reasoning to determine that when comparing fractions with the same denominator, the fraction with the greater numerator is the greater fraction.	**Instructional note:** Throughout this lesson and whenever appropriate, consider reinforcing the idea that to compare fractions we must have the same size whole. If the tasks described in NCTM's Teaching Children Mathematics (2015) article Iteration: Unit Fraction Knowledge and the French Fry Task was done in Topic 12 consider connecting ideas explored in this lesson to revelations from the task. **Solve & Share:** Consider reviewing meaning of the comparison symbols <, >, and =. After introducing the *Solve & Share*, to reinforce the significance of the whole when comparing fractions consider asking student if we have the same size whole (e.g., yes, the whole is 1 mile for both joggers). Then ask if we could compare if they were different size wholes (e.g., no because the wholes have to be the same size in order to compare). **Visual Learning:** Considering pausing and discussing after the question is posed, "Which is greater 4/6 or 2/6?". **Independent Practice/Math Practices and Problem Solving:** Consider discussing item 18 *MP.8 Generalize* to begin to develop schema for the next lesson. ***-continues on next page-***

		Assess & Differentiate: If time permits, you consider teaching students how to play *Tic Tac Toe* (TE, p. 689A). All students should have the opportunity to play this game it provides engaging and meaningful practice of a key concept. Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p. 689A). **Possible Day 2 Solve & Share:** (Read the *Instructional note* at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using item 17 from the *Independent Practice/Math Practices and Problem Solving*. Consider utilizing the following question format during practice: Example 1 Full Statement **Stem:** Which number goes in the box to make the comparison true? $\frac{5}{8} > \frac{\square}{8}$ A. 3 B. 5 C. 7 D. 9 **Rubric:** (1 point) The student selects the correct number (e.g., A).
Lesson 13-4: Use Models to Compare Fraction- Same Numerator		
3.NF.A.3d MP.2 **MP.3** MP.4 MP.6	**Access Prior Learning:** In the previous lesson, students used fraction strips and pictorials to compare fractions with the same denominator. **Developing the Big Idea:** Students are further *developing* fractional sense by comparing fractions with the same numerator. Students use quantitative reasoning to determine that when comparing fractions with the same numerator, the fraction with the greater denominator is less than the other fraction.	**Instructional note:** Throughout this lesson and whenever appropriate, consider reinforcing the idea that to compare fractions we must have the same size whole. If the tasks described in NCTM's Teaching Children Mathematics (2015) article Iteration: Unit Fraction Knowledge and the French Fry Task was done in Topic 12 consider connecting ideas explored in this lesson to revelations from the task. **Visual Learning:** Consider discussing the *Convince Me!* to support student's development of MP.3 Construct Arguments, as well as, to extend the ideas covered in the *Visual Learning Animation*. **Independent Practice/Math Practices and Problem Solving:** Item 18 *Higher Order Thinking* from the *Quick Check* addresses a common misconception that results from students applying reasoning with whole numbers to rational numbers. **Assess & Differentiate:** If time permits, you may consider replacing the *Math and Science Activity* with the game *Display the Digit* (TE, p. 677A), *Tic Tac Toe* (TE, p. 689A), or the *Fluency Practice Activity* (TE, p. 721). Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p. 695A). **Possible Day 2 Solve & Share:** (Read the *Instructional note* at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using item 15 *MP. 3 Critique Reasoning* from the *Independent Practice/Math Practices and Problem Solving*. Consider utilizing the following question format during practice: Example 1 Full Statement **Example Stem:** Select the symbol (< , > , or =) that correctly compares each pair of numbers $\frac{5}{8} \square \frac{5}{6}$ $\frac{3}{6} \square \frac{3}{8}$ | | < | > | = |

Lesson 13-6: Compare Fractions: Use the Number Line

3.NF.A.3d

MP.2
MP.3
MP.6
MP.7

Access Prior Learning:
In lessons 13-1 through 13-4, students used quantitative reasoning and models to compare fractions with either the same numerator or the same denominator. In lesson 3-5, students use quantitative reasoning and benchmark fractions to compare fractions. Students have also represented fractions on a number line.

Developing the Big Idea:
Students further *develop* their fractional sense by comparing fractions using a number line.

Instructional Note:
Lesson 13-6 is recommended to be taught before 13-5 as it offers stronger visual representations for comparing fractions and establishing benchmark fractions which will be explored further in 13-5. Watch for students that are confusing the comparison symbols. Students may have understanding of which fraction is greater than or less than, but still have confusion about which symbol accurately communicates their relationship. Interview these students to determine if they understand the mathematics. If the tasks described in NCTM's Teaching Children Mathematics (2015) article Iteration: Unit Fraction Knowledge and the French Fry Task was done in Topic 12 consider connecting ideas explored in this lesson to revelations from the task.

Solve & Share:
After students have shared their solution methods and reasoning, consider discussing the *Look Back!*

Assess & Differentiate:
If time permits, you may consider replacing the *Math and Science Activity* with the game *Display the Digit* (TE, p. 677A), *Tic Tac Toe* (TE, p. 689A), *Think Together* (TE, p. 701A), or the *Fluency Practice Activity* (TE, p. 721).

Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p. 707A).

Possible Day 2 Solve & Share:
(Read the *Instructional Note* at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using the *Convince Me!*

Lesson 13-5: Compare Fractions- Use Benchmarks

3.NF.A.3d

MP.1
MP.2
MP.3

Access Prior Learning:
In lessons 13-1 through 13-4, students used quantitative reasoning and models to compare fractions with either the same numerator or the same denominator. Students have also represented fractions on a number line.

Developing the Big Idea:
Students are further *developing* fractional sense by comparing fractions using the benchmark fractions 0, $\frac{1}{2}$, and 1 to reason the larger fraction.

Solve & Share:
Consider discussing the *Look Back!* to support students' quantitative reasoning about fractions. If the tasks described in NCTM's Teaching Children Mathematics (2015) article Iteration: Unit Fraction Knowledge and the French Fry Task was done in Topic 12 consider connecting ideas explored in this lesson to revelations from the task.

Visual Learning:
Consider assigning the *Convince Me!* to reinforce ideas shared in the *Visual Learning Animation*.

Assess & Differentiate:
If time permits, consider teaching students how to play *Think Together* (TE, p. 701A). All students should have an opportunity to play this game it provides engaging and meaningful practice of a key concept.

Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p. 701A).

Possible Day 2 Solve & Share:
(Read the *Instructional Note* at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using item 18 *MP. 3 Critique Reasoning* from the *Independent Practice/Math Practices and Problem Solving*.

Consider utilizing the following question format during practice:

Example 1
Full Statement

Stem: Decide whether each comparison is true or false. Click True or False for each comparison.

	True	False
$\frac{3}{4} < \frac{1}{4}$		
$\frac{2}{4} < \frac{2}{3}$		

Rubric: (1 point) The student answers correctly, identifying each as True or False (e.g., F, T).

Lesson 13-7: Whole Numbers and Fractions

3.NF.A.3c
3.NF.A.3a

MP.2
MP.3
MP.7

Access Prior Learning:
In lesson 12-5, students represented fractions on a number line, including fractions greater than 1. In lessons 13-1 and 13-2, students learned that when using fractions to name quantities, a quantity can have more than one equivalent fraction.

Developing the Big Idea:
In this lesson, students are further *developing* their fractional sense by finding that whole numbers can be represented by many different fraction names.

Instructional Note:
Consider making this a 2-day lesson as the **Developing the Big Idea** understanding in this lesson is critical for future grade level work with fractions and has much depth; especially when connecting these understandings to division concepts.

Solve & Share:
To assess student readiness prior to introducing the *Solve & Share*, consider asking students to recall how to represent 1 on the number line when labeling in halves. Consider covering the given model so students are unable to see it and thus more inclined to represent the mathematics using a model that makes the most sense to them.

Visual Learning:
Consider pausing and informally assessing students' connections to Topic 12 and lesson 13-2 when the question is posed, "What are some equivalent fraction names for 1, 2, and 3? How do you know?"

After viewing the *Visual Learning Animation* consider asking the student what is the difference between $\frac{1}{2}$ and $\frac{2}{1}$. and to create a model to prove the difference.

Assess & Differentiate:
If time permits, you may consider replacing the *Problem Solving Reading Mat* with the game *Display the Digit* (TE, p. 677A), *Tic Tac Toe* (TE, p. 689A), *Think Together* (TE, p. 701A), or the *Fluency Practice Activity* (TE, p. 721).

Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p. 713A).

Possible Day 2 Solve & Share:
(Read the *Instructional Note* at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using items 18 *Higher Order Thinking* from the *Homework & Practice.*

Consider utilizing the following question formats during practice:

Example Stem 1:
What denominator goes in the box (☐) to make the equation true?

$2 = \frac{2}{\square}$

Enter your answer in the response box.

Rubric: (1 point) The student enters the correct value (e.g., 1).

Example Stem 2:
What numerator goes in the box (☐) to make the equation true?

$\frac{\square}{1} = 2$

Enter your answer in the response box.

Rubric: (1 point) The student enters the correct value (e.g., 2).

Example Stem 1:
What numerator goes in the box (☐) to make the equation true?

$\frac{\square}{2} = 1$

Rubric: (1 point) The student enters the correct value (e.g., 2).

Example Stem 2:
What denominator goes in the box (☐) to make the equation true?

$1 = \frac{2}{\square}$

Rubric: (1 point) The student enters the correct value (e.g., 2).

Lesson 13-8: Math Practices and Problem Solving- Construct Arguments		
3.NF.A.3b **3.NF.A.3d** **MP.3** MP.1 MP.4 MP.5 MP.6	**Access Prior Learning:** In previous lessons, students have developed an understanding that when using fractions to name quantities, a quantity can have more than one equivalent fraction. **Securing the Big Idea:** Students are further *securing* their understanding of equivalent fractions and comparing fractions by constructing arguments to solve problems in real-world contexts.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 3. Refer to the *Math Practices and Problem Solving Handbook* (TE, pp. F23-F23A, F29) for suggestions on how to develop, connect and assess this Math Practice. **Solve & Share:** Consider reintroducing MP. 3 Thinking Habits (SE, p. F23) before introducing the *Solve & Share*. Also consider using time students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.3 that are listed in the *Math Practices and Problem Solving Handbook* (TE, p. F23A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice. **Solve & Share:** After discussing students' solution methods and reasoning, consider discussing the *Look Back!* prompt if these ideas did not already come out from the classroom discussion. **Visual Learning:** Consider pausing to discuss," Why are 2 number lines a good drawing to justify the conjecture?" and, "Is this a representation you might try to use? Why or why not?" **Assess & Differentiate:** If time permits, consider teaching students how to play the game *Teamwork* (TE, p. 719A). Child-watch to identify students who need additional support and consider the Intervention Activity provided (TE, p. 719A).

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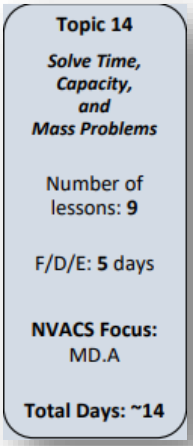
Grade 3 Topic 14: Solve Time, Capacity, and Mass Problems



3rd Grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Measurement and Data (Measurement Part) (pp. 16-18)

Prior to instruction, view the Topic 14 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 733A-733F), the Topic Planner (pp.733I-733K), all 9 lessons, and the Topic Assessments (pp. 803-804A).

Mathematical Background: Read Topic 14 Cluster Overview/Math Background (TE, pp. 733A-733F)	**Topic Essential Question:** How can time, capacity, and mass be measured and found? *Reference Answering the Topic Essential Question (TE, pp. 799-800) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

14-1	14-2	14-3	14-4	14-5	14-6	14-7	14-8	14-9	Assessment

5 F/D/E days used strategically throughout the topic.

Instructional note:

Topic 14's big idea is that some attributes of objects are measurable and can be quantified using unit amounts. In this topic students learn to solve a variety of problems involving measurement the attributes of time, capacity (liquid volume), and mass. The Measurement Data (Measurement Part) progression document states that working problems with intervals of time, liquid volumes, and masses of objects supports, "the work done in multiplication and the mathematical practices of making sense of problems (SMP 1) and representing them with equations, drawings, or diagrams" (NVACS, 2010, SMP 4).

In Topic 14, students first investigate time by extending the second grade understanding from standard 2.MD.C.7, "Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m." (NVACS, 2010). In third grade, students "Tell and write time to the nearest minute and measure time intervals in minutes and solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram" (NVACS, 2010, 3.MD.A.1). In Topic 14, students will develop this understanding by finding that:

- Time can be measured using seconds, minutes, and hours. Lengths of time can be found by adding or subtracting time intervals.
    - There is more than one way to write and tell time to the nearest minute. We can use analog clocks, digital clocks, words, numbers, and symbols to show and tell time as precisely as possible (TE, p. 740).
    - When solving for elapsed time it is helpful to first count the number of elapsed hours and then the number of elapsed minutes (TE, p. 746). Sometimes, the elapsed time is provided, and students need to figure out the start or end time.
    - The methods used to solve elapsed time questions are like those used to solve for other addition and subtraction problems (TE, p.752).

Another unit of measure that students will be exploring in Topic 14 is capacity. Capacity is the amount a container can hold measured in liquid units. To help students develop benchmark measurements in capacity and to make concepts more concrete, consider providing students with concrete experiences with measuring liquids. In Topic 14 students will be introduced to milliliters and liters as metric units for measuring capacity. Students may need support on the conventions of reading the markings in a 1-liter container or graduated cylinder. Again, consider introducing this with concrete models (e.g., graduated cylinders), if available, before having students read measurements from a pictorial representation. Measurement Data (Measurement Part) progression document states that:

> "Compared to the work in area, volume introduces more complexity, not only in adding a third dimension and thus presenting a significant challenge to students' spatial structuring, but also in the materials whose volumes are measured. These materials may be solid or fluid, so their volumes are generally measured, e.g., "packing" a right rectangular prism with cubic units or "filling" a shape such as a right circular cylinder" (2012, p.19).

Finally, students will be exploring mass as a unit of measurement. Mass is the amount of matter in an object. Metric units for measuring mass include grams and kilograms. In this topic, students will develop understanding that knowing benchmark measurements of mass is helpful in estimating the mass of other objects. Students will also come to realize that one way to measure

the mass of an object is to use a pan balance and metric weights. Please note that the measure of mass is different from weight. Mass is a measurement of the amount of matter something contains, while weight is the measurement of the pull of gravity on an object.

While teaching this topic you may want to consider providing experiences and facilitating discussions that help students to develop benchmark measurements for the different units of measure explored. This is often accomplished by providing concrete learning experiences with the units of measure and comparing new units of measure to known measurements. For example, students that are comfortable with the size of a measuring cup can visually see how many milliliters are equivalent to the cup and then decide what others amounts would be appropriate to measure with milliliters. As much as possible, experiences with the following materials are recommended to facilitate students developing benchmark measurements:

- Pan balance
- Metric weights
- Gram weights
- Kilogram weights
- 1-Liter bottle
- Large bowls
- Eye dropper or pipette
- Gallon container
- Graduated cylinder
- 1-Liter beaker

Focus Math Practice 2: Reason abstractly and quantitatively
The Nevada Academic Content Standards state that, "Mathematically proficient students make sense of quantities and their relationships in problem situations. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects" (NVACS, 2010, SMP 2). Focus on opportunities for students to develop Mathematical Resources to support students' development of MP. 2 include the Teacher's Edition (pp. F22 - F22A) and the Nevada Academic Content Standards for Mathematical Practice.

Looking ahead to the Topic Performance Assessment, students need select an appropriate units of measure and be able to use a number line to represent elapsed time.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
time interval volume capacity (liquid volume) millimeter liter mass gram kilogram	*estimate* *analog clock* *minute hand* *hour hand* *elapsed time* *A.M.* *P.M.*

Additional terminology that students may need support with: *clock face, nearest, past, volume, abbreviations*

***Collaborative Team Conversations (CTC)**
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

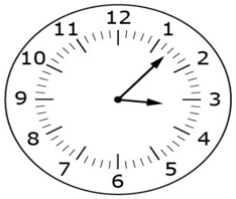
Guiding questions: "Are students able to determine the elapsed time?"
"Are students able to use their knowledge of operations to solve real world mass and liquid volume problems?

Lesson	Evidence	Look for
14-2	***Quick Check*** (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources". • students understand time as a measurement (elapsed time)
14-8	***Solve & Share*** (student work samples)	Focus CTC around the big idea: • student strategies and models

		• use of operational knowledge to solve mass and/or liquid volume problems

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 799-804	Use *Scoring Guide* TE pp. 799-804A

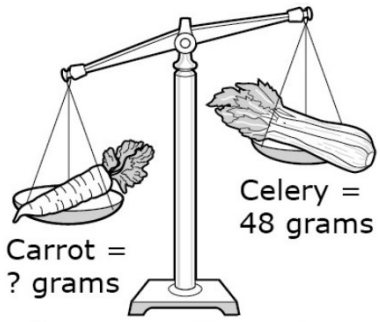
Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	**Mathematical Development of the Big Idea**	**Instructional Clarifications & Considerations**
Lesson 14-1: Time to the Minute		
3.MD.A.1 MP.3 MP.4 MP.5 MP.6	**Access Prior Learning:** In Topic 8, Grade 2, students learned to tell time to the nearest 5 minutes. In Topic 2, Grade 3, students learned about patterns with 5 as a factor. **Developing the Big Idea:** Students are further *developing* time concepts by applying their knowledge of counting by 5s and 1s to tell time to the nearest minute.	**Topic Opener:** Introduce the *Topic Essential Question*, "How can time, capacity, and mass be measured and found?". Consider using this question to begin a class anchor chart to which new ideas can be added each day. This allows students to see the development of their own thinking and ideas and make new connections with the content of this topic. Consider having students complete the *Review What You Know* prior to beginning instruction on Topic 14 so that you can respond to students' instructional needs using the *Item Analysis for Diagnosis and Intervention* (TE, p. 734). Consider introducing vocabulary terms as they are encountered in the lessons rather than introducing all terms at the beginning of the topic. **Solve & Share:** Consider having tools readily available for students to use such as clocks or Teaching Tool 20. Watch for students who struggle to begin the *Solve & Share*. Ask students what the tic marks on the clock represent to help them connect to the work they did in 2nd grade. Consider having students share their solution methods and reasoning that match the samples provided. Both Jasmin & Timothy's work correctly marks the minutes; however, only Jasmin's work accurately notes where the hour hand would be for the given time. Showing both of these solution methods provides the opportunity to discuss movement of the hour hand. Consider wrapping up the whole class discussion by asking students how they can tell time to the nearest minute. The *Visual Learning Animation* can then be used to confirm, clarify or correct students' ideas. **Visual Learning:** Consider pausing and discussing after it poses the question, "Why is an analog clock a good tool for showing time to the nearest minute?" Use tools to have students show time in different ways. For example, contrasting the difference between digital and analog time. **Assess and Differentiate:** If time permits, teach students how to play *Display the Digit* (TE, p. 743A). All students should have the opportunity to play this game. Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 743A). Consider utilizing the following question format during practice: Example Stem: Use this clock to answer the question.  Select the time to the nearest minute, shown on the clock. A. 1:15 B. 2:07 C. 3:07 D. 7:35

Lesson 14-2: Units of Time- Measure Elapsed Time		
3.MD.A.1 MP.1 **MP.2** MP.3	**Access Prior Learning:** In Topic 14, Grade 2, students learned about elapsed time. In Topic 8, Grade 2, students were introduced to the abbreviations A.M and P.M. In the previous lesson, students learned to tell time to the nearest minute. **Developing the Big Idea:** Students further *develop* time concepts by finding elapsed time in 1-hour and 5-minute intervals.	**Solve & Share:** Consider having blank clock faces available (Teaching Tool 20). After introducing the *Solve & Share*, consider asking the questions provided in *Build Understanding* to ensure all students are problem solving with the same constraints of the problem. **Look Back:** After students have shared their solution methods and reasoning, consider discussing the *Look Back!* prompt to build students' estimation skills with time. **Visual Learning:** Consider pausing the *Visual Learning Animation* to discuss the questions, "How can you find elapsed time?" and "How long did the walk last?" **Convince Me:** Consider discussing the *Convince Me!* prompt to confront confusion that often occurs regarding A.M. and P.M times with elapsed time. **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 12 to provide distributed practice of "Put Together/Addend Unknown" (see page 88 of NVACS, 2010 for more information of problem types) problem type and algebraic reasoning. **Assess and Differentiate:** If time permits, teach students how to play *Clip and Cover* (TE, p. 749A). All students should have the opportunity to play this game. Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 749A). ***CTC: *Quick Check*** (digital platform)
Lesson 14-3: Units of Time- Solve Word Problems		
3.MD.A.1 MP.1 **MP.2** MP.3 MP.4	**Access Prior Learning:** In previous lessons, students told time to the nearest minute and measured elapsed time. **Developing the Big Idea:** Students are further *developing* time concepts by solving problems that involve addition and subtraction of time intervals and using the number line or bar diagram to model the math.	**Instructional note:** Students should be allowed to choose whether they would like to model the addition or subtraction of the time intervals with a bar diagram or a number line. However, while students work on the *Solve & Share*, they should be allowed to solve with any model/method they choose. **Solve & Share:** After introducing the *Solve & Share*, consider asking the questions provided in *Build Understanding* to ensure all students are problem solving with the same constraints of the problem. **Independent Practice/Math Practices and Problem Solving:** Consider discussing item 8 to provide students distributed practice with representing whole numbers as fractions and the opportunity to reinforce fractional number sense development. **Assess and Differentiate:** If time permits, you consider replacing the *Problem Solving Reading Mat* with the games *Display the Digit* (TE, p. 743A), *Clip and Cover* (TE, p. 749A), or the *Fluency Practice Activity* (TE, p. 793). Child watch to identify students who need additional support and pull them in a small group to do the *Intervention Activity* (TE, p. 755A). Consider utilizing the following question format during practice: Example 1 Full Statement **Example Stem:** A music class starts at 1:32 p.m and ends at 2:15 p.m. Enter the length, in minutes, of the music class, in the response box. **Rubric:** (1 point) The student correctly enters the length of the class in minutes (e.g., 43).

Lesson 14-4: Estimate Liquid Volume		
3.MD.A.2 MP.1 **MP.2** MP.4 MP.6 MP.8	**Access Prior Learning:** In Topic 8, Grade 3, students learned to estimate sums and differences. **Developing the Big Idea:** Students further *develop* an understanding of estimation and units of measurement by developing benchmarks to estimate capacity (liquid volume).	**Instructional note:** A 1-liter bottle and large bowls are necessary for the *Solve & Share* to help students develop benchmark measurements. If available, consider also having a gallon container to address ideas introduced in the *Visual Learning Animation*. Consider reading the *Coherence* section for more suggestions on developing benchmark measurements of milliliters (TE, p. 757A). **Solve & Share:** After introducing the *Solve & Share,* consider asking the questions provided in *Build Understanding* to ensure all students are problem solving with the same constraints of the problem. **Look Back:** After students have shared their solution methods and reasoning, consider discussing the *Look Back!* prompt. **Visual Learning:** Consider discussing the ideas provided in the *Prevent Misconceptions* section (TE, p. 758) during the *Visual Learning Animation*. **Convince Me:** To support students' development of MP. 2 consider discussing the *Convince Me!* prompt. **Independent Practice/Math Practices and Problem Solving:** Consider discussing item 18 to help students develop familiarity with liters (L) and milliliters (mL) to be able to select the appropriate unit of measure for the items listed. **Assess and Differentiate:** If time permits, teach students how to play *Toss and Talk* (TE, p. 761A). All students should have the opportunity to play this game. Child watch to identify students who need additional support and pull them in a small group to do the *Intervention Activity* (TE, p. 761A).
Lesson 14-5: Measure Liquid Volume		
3.MD.A.2 MP. 3 MP.4 MP.5 MP.6 MP.8	**Access Prior Learning:** In the previous lesson students developed benchmarks to estimate capacity (liquid volume) in L and mL. **Developing the Big Idea:** Students further *develop* an understanding of estimation and units of measure by using standard units, liters (L) and milliliters (mL), to estimate capacity (liquid volume).	**Instructional note:** A marked 1-liter beaker and 6 containers are necessary to complete the *Solve & Share* and to help students develop benchmark measurements. **Look Back:** Consider discussing the *Look Back!* to support students' problem solving and appropriate use of mathematical tools after students have had an opportunity to work on the *Solve & Share*. **Visual Learning:** Consider pausing and discussing the following questions from the *Visual Learning Animation*: • "How can he find the capacity of the fish bowl?" • "What does the abbreviation mL mean?" • "How many mL are represented by each little mark on the 1-Liter container?" • "If the top mark were labelled in mL, what would it say?" **Convince Me:** Consider discussing the *Convince Me!* to support students' development of benchmark measurements and choosing an appropriate unit of measure. **Assess and Differentiate:** If time permits, consider teaching students how to play *Teamwork* (TE, p. 767A). All students should have an opportunity to play this game. Based upon child-watching, identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 767A).
Lesson 14-6: Estimate Mass		
3.MD.A.2 **MP.2** MP.3 MP.4	**Access Prior Learning:** In Topic 8, Grade 3, students learned to estimate sums and differences. Students have also applied estimation skills to measurement throughout this topic.	**Instructional note:** If available, have a pan balance, gram, and kilogram weights to help students develop benchmark measurements. To support students' understanding of mass and their development of MP. 6, "Attend to precision" watch for students that confuse weight and mass. Consider correcting inaccurate language of describing an object as "weighing" some amount by providing the language "has the mass of" to describe the object. ***-continues on next page-***

MP.5	**Developing the Big Idea:** Students further *develop* an understanding of units of measure by finding that mass is a measure of the quantity of matter in an object. Students also further *develop* understanding of units of measure by estimating mass measurements in grams and kilograms.	**Assess and Differentiate:** If time permits, you may consider replacing the *Math and Science Activity* with the game *Display the Digit* (TE, p. 743A), *Clip and Cover* (TE, p. 749A), *Toss and Talk* (TE, p. 761A), *Teamwork* (TE, p. 767A), or the *Fluency Practice Activity* (TE, p. 793). Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 773A).
Lesson 14-7: Measure Mass		
3.MD.A.2 MP.1 **MP.2** MP.3 MP.6 MP.7	**Access Prior Learning:** In the previous lesson, students learned about estimating mass. **Developing the Big Idea:** Students further *develop* understanding of estimation and units of measure by using standard units, grams (g) and kilograms (Kg), to estimate mass.	**Instructional note:** A pan balance, metric weights, and assorted objects are necessary to complete the *Solve & Share* and to help students develop benchmark measurements. **Look Back:** After student solution methods and reasoning have been shared, consider using the *Look Back!* to facilitate a class discussion. **Convince Me:** To support students' reasoning with the appropriate unit of measure, consider discussing the *Convince Me!* **Assess and Differentiate:** If time permits, you may consider replacing the *Problem Solving Reading Mat* with the game *Display the Digit* (TE, p. 743A), *Clip and Cover* (TE, p. 749A), *Toss and Talk* (TE, p. 761A), *Teamwork* (TE, p. 767A), or the *Fluency Practice Activity* (TE, p. 793). Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 779A).
Lesson 14-8: Solve Word Problems Involving Mass and Liquid Volume		
3.MD.A.2 MP.1 **MP.2** MP.4 MP.6	**Access Prior Learning:** In previous lessons in this topic, students have estimated and measured for capacity and mass. **Developing the Big Idea:** In this lesson, students are *developing* an understanding of measuring capacity and mass by using all four operations to solve problems.	**Visual Learning:** During the *Visual Learning Animation*, consider doing the *Try It!* activity as this connects the context of the problem to the visual representations and model. After the *Try It!*, the *Visual Learning Animation* makes the connections between the context and the bar diagram explicit. Consider pausing the video after it shows the connections to discuss what is known and unknown, what operation is needed to solve, and how to complete the bar diagram. **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 9 to provide students distributed practice with elapsed time. On *Quick Check* item 12 *Common Core Assessment*, watch for students that have incorrect responses as a result of struggling to read the pictorial representations of the containers. **Assess and Differentiate:** If time permits, consider teaching students how to play *Teamwork* (TE, p. 785A). All students should have an opportunity to play this game. Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 785A). ***CTC: *Solve & Share*** (student work samples) ***-continues on next page-***

		Consider utilizing the following question during practice: Example 1 Full Statement **Example Stem 1:** A bunch of celery has a mass of 48 grams. A carrot has a mass that is 15 grams more than the celery.  Enter the mass, in grams, of the carrot, in the response box. **Rubric:** (1 point) The student writes the correct solution (e.g., 63). Example 2 Full Statement **Example Stem 2:** A farmer takes 46 kilograms of potatoes to the market. The farmer sells 29 kilograms of the potatoes. Enter the number of kilograms of potatoes the farmer has left in the response box. **Rubric:** (1 point) The student writes the correct solution (e.g., 17).
Lesson 14-9: Math Practices and Problem Solving- Reasoning		
3.MD.A.1 **MP.2** MP.1 MP.3 MP.4 MP.6 MP.8	**Access Prior Learning:** In previous lessons, students solved problems involving time. **Developing the Big Idea:** Students are *developing the* understanding of elapsed time by making sense of the quantities and relationships to solve problems in real-world contexts.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 2. Refer to the *Math Practices and Problem Solving Handbook* for suggestions on how to develop, connect and assess this Math Practice (TE, pp. F22-F22A, F29). Also reference the handbook in the Student Edition (SE, p. F22). **Solve & Share:** Consider reintroducing MP. 2 Thinking Habits (SE, p. F22) before introducing the *Solve & Share*. Also consider using the time students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.2 that are listed in the *Math Practices and Problem Solving Handbook* (TE, p. F22A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice. **Look Back:** After discussing students' solution methods and reasoning, consider discussing the *Look Back!* prompt to support students understanding of when a time when be the answer versus minutes being the answer to elapsed time problems. **Convince Me:** To support students' development of MP. 2, consider discussing the *Convince Me!* prompt. **Assess and Differentiate** If time permits, you may consider replacing *the Math and Science Activity* with the game *Display the Digit* (TE, p. 743A), *Clip and Cover* (TE, p. 749A), *Toss and Talk* (TE, p. 761A), *Teamwork* (TE, p. 767A), *Teamwork* (TE, p. 785A), or the *Fluency Practice Activity* (TE, p. 793). Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 791A).

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Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Van De Walle, J. A., Bay-Williams, J. M., Lovin, L. H., & Karp, K. S. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 3-5* (2nd ed.). New York, NY: Pearson.

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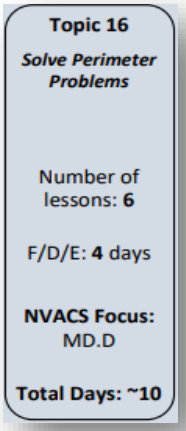
Grade 3 enVision Topic 16: Solve Perimeter Problems



3rd Grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Measurement and Data (Measurement Part) (pp. 16-18)

Prior to instruction, view the Topic 16 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 843A-843F), the Topic Planner (pp.843I-843J), all 6 lessons, and the Topic Performance Assessment (pp. 889-890A).

Mathematical Background: Read Topic 16 Cluster Overview/Math Background (TE, pp. 843A-843F)	**Topic Essential Question:** How can perimeter be measured and found? *Reference Answering the Topic Essential Question (TE, pp. 887-888) for key elements of answers to the Essential Question.*

The lesson map for this Topic is as follows:

16-1	16-2	16-3	16-4	16-5	16-6	Assessment

4 F/D/E days used strategically throughout the topic.

Instructional note:
Topic 16's big idea is that some attributes of objects are measurable and can be quantified using units. New learning to this topic is perimeter as a measurable unit, while developing a deeper understanding of area through exploring the relationship between area and perimeter.

Perimeter as defined by the Geometric Measurement progression document, "is the boundary of a two-dimensional shape. For a polygon, the length of the perimeter is the sum of the lengths of the sides" (2012, p.16). Students begin to develop understanding of perimeter concepts by finding the perimeter of polygons on a grid. A common misconception when determining the perimeter of shapes on a grid is to count the vertices rather than the unit segments. In such cases, support students by clarifying what/how to count the unit segments to determine the side lengths. See the *Math Background* pages for information regarding this.

Students further develop understanding of perimeter concepts by determining the perimeter of parallelograms when only 2 lengths of adjacent sides are provided. Students may choose to solve by doubling each side's length and adding them together or by adding the adjacent sides' measures and doubling. A common error in these cases is for students to only add the 2 side lengths where the measures are given. In this event, revisit the definition of perimeter and ask students what sides they have found the total length for and what they need to find to get the perimeter of the shape.

In the case where a parallelogram is a square, only 1 side's length may be offered and students will have to reason with what they know about attributes of squares to determine the perimeter of a square. In this case students may choose to add the measure 4 times (repeated addition), double the measure and double it again (a strategy for solve for multiplication facts with 4 as a factor), or multiply the length of the one side times 4 (or 4 times). To connect to previous learning this year, and to revisit understandings of grade 3 critical content area of multiplication, it may be a worthy class discussion on these 3 different solution strategies and why they all work and in what situations or context one may work better than another.

Students also develop understanding of perimeter concepts by exploring how to find perimeter when they have to solve for an unknown side length of polygons. Initially **enVision**math**2.0** represents the unknown side length with a question mark (?); however, further in the lesson it is represented with a variable and the unit of measurement. For example, if side lengths were measured in centimeters they identify the unknown side length as "*x* cm." This connects with standard 3.OA.D.8, "Solve 2-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity" (NVACS, 2010). You may need to clarify the difference between the letter for the unknown and the letters for the abbreviated unit of measurement.

Students also explore the relationship between area and perimeter. Students often confuse perimeter and area measures. The Geometric Measurement progression document states that, "Differentiating perimeter from area is facilitated by having students draw congruent rectangles and measure, mark off, and label the unit lengths all around the perimeter on one rectangle, then do the same on the other rectangle but also draw the square units. This enables students to see the units involved in length and area and find patterns in finding the lengths and areas of non-square and square rectangles (MP 7)" (2012, p. 18). Chapin and Johnson (2006) suggest asking the following questions to facilitate students developing understanding of the relationship between area and perimeter:

- What do all the figures with smaller perimeters have in common?
  - The figures with smaller perimeters are more condensed and compact.
  - The shape of these figures is more closely related to a square.
- What do all the figures with large perimeters have in common?
  - The figures with larger perimeters are elongated. Most of the square tiles are adjacent to another square tile only on one side (p.284-285).

Students do not need to be secure in the responses to these questions, the questions just help to identify that there is a relationship between perimeter and area.

Focus Math Practice 2: **Reason abstractly and quantitatively**
Focus opportunities for students to develop Mathematical Practice 2 behaviors throughout the entire topic, as this is the focus of the Math Practices and Problem Solving lesson 16-6. Resources to support students' development of MP. 2 include the Teacher's Edition (pp. F22 - F22A) and the Nevada Academic Content Standards for Mathematical Practice. The Nevada Academic Content Standards state that, "Mathematically proficient students make sense of quantities and their relationships in problem situations. … Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects" (2010, SMP 2).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
perimeter equilateral triangle	*area* *square units* *unit square* *scale (multiplicative scale)*

Additional terminology that students may need support with: *grid, distance, around, representations,*

***Collaborative Team Conversations (CTC)**
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

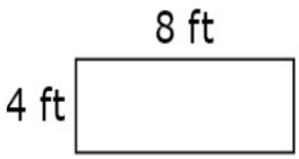
Guiding questions: "Are students showing understanding that rectangles can have the same area and different perimeters?"
"Are students able to explain the relationship between area and perimeter using rectangles with the same area and different perimeters?"

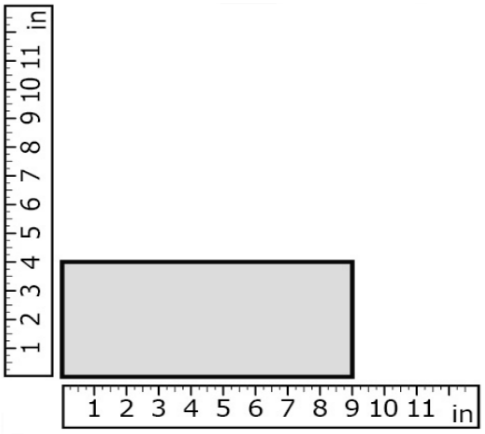
Lesson	Evidence	Look for
16-2	***Quick Check*** (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources". • using given measurements to determine the perimeter of a polygon.
16-5	***Solve and Share*** (student work samples)	Focus CTC around the big idea: • comparing the relationship between the area and perimeter of rectangles.

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 887-890	Use *Scoring Guide* TE pp. 887-890A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 16-1: Understand Perimeter		
3.MD.D.8 MP.1 **MP.2** MP.3 MP.4 MP.6	**Access Prior Learning:** In Topic 6, Grade 3, students learned how to find area using standard, as well as nonstandard, units of measurement. Students also learned how to count unit squares to determine side length. **Developing the Big Idea:** Students *begin* to understand perimeter as the distance around a figure and solve for perimeter.	**Topic Opener:** Introduce the *Topic Essential Question*, "How can perimeter be measured and found?" (TE p. 843). Consider using this question to begin a class anchor chart to which new ideas can be added each day. This allows students to see the development of their own thinking and ideas and make new connections with the content of this topic. Consider having students complete the *Review What You Know* prior to beginning instruction on Topic 16 so that you can respond to students' instructional needs using the *Item Analysis for Diagnosis and Intervention* (TE, pp. 844-846). Consider introducing vocabulary terms as they are encountered in the lessons rather than introducing all terms at the beginning of the topic. **Solve & Share:** Watch for students that count vertices rather than the unit segments. For ideas on supporting students that are miscounting the unit segments read *Prevent Misconceptions* (TE, p. 848). ***-continues on next page-***

		Visual Learning: Consider pausing the *Visual Learning Animation* after it asks, "How do you find the perimeter?" The *Visual Learning Animation* introduces finding perimeter using side measurements without the grid. Consider asking students "How does removing the grid lines change finding the perimeter?" Consider assigning the *Convince Me!* to support students' development of perimeter measurements. **Independent Practice/Math Practices and Problem Solving:** Consider assigning *Homework and Practice* item 8 to give students the opportunity to create a polygon using a given perimeter measurement. **Assess & Differentiate:** If time permits, you may consider using the *Math and Science Activity*. Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 851A).
Lesson 16-2: Perimeter of Common Shapes		
3.MD.D.8 MP.1 **MP.2** MP.3 MP.6 MP.7 MP.8	**Access Prior Learning:** In Topic 15, Grade 3, students learned about sides of polygons as being attributes of polygons. In the previous lesson, students found the perimeter of polygons by counting the unit segments around a figure or adding all the sides' measurements. **Developing the Big Idea:** Students *develop* perimeter concepts by using reasoning and their knowledge of attributes of polygons, to finding the perimeter of figures with missing side lengths.	**Solve & Share:** Watch for students that appear to be struggling, help them to apply knowledge of the attributes of rectangles. Support students' problem solving by asking them what they know about rectangles (e.g., opposites sides are the same length, 2 pair of parallel sides, 4 right angles, etc.). Then ask them which of those attributes could help them figure out the perimeter of the rectangle (e.g., opposites sides are the same length). After students have shared their solution methods and reasoning, consider discussing the *Look Back!* to help students focus on how they can use multiplication and addition to find perimeter. **Visual Learning:** After the *Visual Learning Animation* show how repeated addition can be used to solve for the perimeter of a square. Consider asking, "Is there another way to solve for the perimeter of squares?" Are students connecting using repeated addition to multiplication to find perimeter? **Assess & Differentiate:** If time permits, teach students how to play *Clip and Cover* (TE, p. 857A). All students should have the opportunity to play this game. Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 857A). ***CTC: *Quick Check*** (digital platform) Consider utilizing the following question format during practice: Example 2 Full Statement **Example Stem 2:** Ms. Smith needs to find the perimeter of her rectangular garden. She wants to put a fence around her entire garden. Her garden measures 8 feet by 4 feet as shown.  Enter the perimeter, in feet, of the garden in the response box. **Rubric:** (1 point) The student correctly enters the perimeter of the shape (e.g., 24). **-continues next page-**

		Example 3 Full Statement **Example Stem 3:** The rulers give the measurement for two sides of the rectangle.  Enter the perimeter, in inches, of the rectangle in the response box. **Rubric:** (1 point) The student correctly enters the perimeter of the shape (e.g., 26).
Lesson 16-3: Perimeter and Unknown Side Lengths		
3.MD.D.8 MP.1 **MP.2** MP.3 MP.4 MP.7	**Access Prior Learning:** In the previous lessons, students solved for perimeter by using attributes of regular polygons where some of the side lengths were not labelled. **Developing the Big Idea:** Students are further *developing* perimeter concepts by solving for a missing side length in a polygon with a given perimeter.	**Solve & Share:** Watch for students that incorrectly determine the unknown side as 3 ft. They are likely applying reasoning based on understanding rectangles. Remind students what is known and unknown in this problem. Can they write an equation that represents the known sides, the unknown side (use a variable) and perimeter? How does this expression help to determine the missing side length? Are students able to use an inverse operation to check their solution? **Visual Learning:** During the *Visual Learning Animation* consider pausing to allow students to solve for *x* in the equation *x* + 18 = 22. Are they able to use more than one operation to find the value of x? **Assess & Differentiate:** If time permits, consider having student play *Clip and Cover* (TE, p. 857A) or the *Fluency Practice Activity* (TE p. 883). Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 863A).
Lesson 16-4: Same Perimeter, Different Area		
3.MD.D.8 **3.MD.C.7b** MP.1 **MP.2** MP.3 MP.6 MP.7 MP.8	**Access Prior Learning:** In previous lessons, students found the perimeter of polygons, in some cases, with an unknown side length. **Developing the Big Idea:** Students further *develop* perimeter and area concepts by using what they know about perimeter and rectangles to discover that polygons with the same perimeter can have different areas.	**Solve & Share:** To assess student readiness, consider asking students, "What is the difference between perimeter and area?" Are students able to describe area as the measure of space inside a figure and perimeter as the measure of the distance around a figure. Also consider asking students "What is the same?" to remind them that both are measurements. Adding these ideas to the class anchor chart will allow students to revisit these concepts in future lessons. After students have shared their solution methods and reasoning, use the *Look Back!* to extend thinking about the relationship between perimeter and area. **Visual Learning:** Consider pausing the *Visual Learning Animation* and giving students time to solve for the area of the shapes. *Prevent Misconceptions* section (TE, p. 866) suggests checking to be sure that students understand how to find perimeter and area. Can students generalize how side length is related to area? Also, consider discussing the *Convince Me!* to extend upon ideas presented in the *Visual Learning Animation*. **Independent Practice/Math Practices and Problem Solving:** Consider assigning *Math Practices and Problem Solving* item18 to provide students distributed practice with a division situation. **Assess & Differentiate:** If time permits, teach students how to play *Teamwork* (TE, p. 869A). All students should have the opportunity to play this game. Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 869A).

Lesson 16-5: Same Area, Different Perimeter

3.MD.D.8
3.MD.C.7b

MP.1
MP.2
MP.3
MP.4
MP.5
MP.7
MP.8

Access Prior Learning:
In the previous lesson, students discovered that different rectangles can have the same perimeter and different areas.

Developing the Big Idea:
Students further *develop* perimeter and area concepts by using what they know about area and rectangles to discover that polygons with the same area can have different perimeters.

Solve & Share:
Prior to introducing the *Solve & Share* consider having centimeter grid paper (Teacher Tool 13) and colored tiles available. After introducing the *Solve & Share,* ask students what tool might be helpful in solving the problem.

After students have shared their solution methods and reasoning, consider discussing the *Look Back!* if the relationship between area and perimeter was not yet discussed.

Visual Learning:
Consider pausing and discussing the *Visual Learning Animation* after it asks the question, "Why do the water tiles surround this rectangle?"

Consider discussing the *Convince Me!* to support students' development with choosing the appropriate unit of measure (linear vs. square units).

Assess & Differentiate:
If time permits, consider using the *Math and Science Activity*, games *Clip and Cover* (TE, p. 857A), *Teamwork* (TE, p. 869A), or the *Fluency Practice Activity* (TE, p. 883).

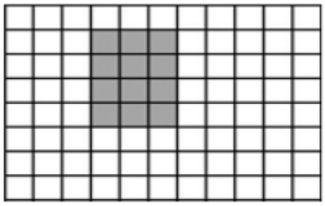
Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 875A).

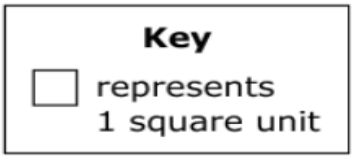
CTC: *Solve and Share (student work samples)

Consider utilizing the following question format during practice:

Example 1
Full Statement

Example Stem: A shaded rectangle is shown on the grid.





Part A: What is the perimeter, in units, of the shaded rectangle?
Enter your answer in the first response box.

Part B: What is the area, in square units, of the shaded rectangle?
Enter your answer in the second response box.

Lesson 16-6: Math Practices and Problem Solving- Reasoning

3.MD.D.8

MP.2
MP.1
MP.3
MP.6
MP.7

Access Prior Learning:
In previous lessons, students developed perimeter concepts and learned about the relationship between perimeter and area.

Developing the Big Idea:
Students are *developing* understanding of perimeter concepts by focusing on MP.2 to understand the relationship between numbers in order to simplify and solve problems involving perimeter.

This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 2. Refer to the *Math Practices and Problem Solving Handbook* for suggestions on how to develop, connect and assess this Math Practice (TE, pp. F22-F22A, F29). Also reference the handbook in the Student Edition (SE, p. F22).

Solve & Share:
Consider reintroducing MP. 2 Thinking Habits (SE, p. F22) before introducing the *Solve & Share*. Also consider using the time students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.2 that are listed in the *Math Practices and Problem Solving Handbook* (TE, p. F22A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice.

Visual Learning:
To support students' development of MP. 2, consider discussing the *Convince Me!* prompt.

Assess & Differentiate:
Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 881A).

References

Chapin, S. H., & Johnson, A. (2006). *Math matters: Understanding the math you teach, Grades K-8*. Sausalito, CA: Math Solutions Publications.

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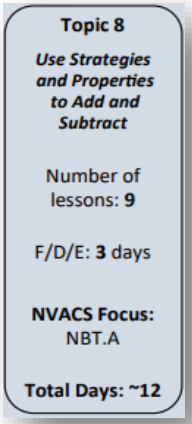
Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Van De Walle, J. A., Bay-Williams, J. M., Lovin, L. H., & Karp, K. S. (2014).*Teaching student-centered mathematics: Developmentally appropriate instruction for grades 3-5* (2nd ed.). New York, NY: Pearson.

3rd Grade Topic 8: Use Strategies and Properties to Add and Subtract



[3rd Grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: Numbers and Operations in Base Ten, K-5 (p. 12)

Prior to instruction, view the Topic 8 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 401A-401F), the Topic Planner (pp.401I-401K), all 9 lessons, and the Topic Assessments (pp. 469-470A).

Mathematical Background: Read Topic 8 Cluster Overview/Math Background (pp. 401A-401F)	**Topic Essential Question:** How can sums and differences be estimated and found mentally? *Reference Answering the Topic Essential Question (TE, pp. 465-466) for key elements of answers to the Essential Question.*

The lesson map for this Topic is as follows:

8-1	8-2	8-3	8-4	8-5	8-6	8-7	8-8	8-9	Assessment

3 F/D/E days used strategically throughout the topic.

Instructional note:

Topics 8, 9 and 10 comprise a cluster on understanding place-value and the properties of operations to perform multi-digit operations. Topic 8 focuses on the properties of operations for addition: Commutative (Order) Property, Associative (Grouping) Property and Additive Identity Property of 0 (Zero). *Note: The Additive Identity Property for 0 (3+**0**=3) is different than The Multiplicative Identify Property for 1 (3 x **1** = 3) and unless explicitly discussed and compared may cause confusion for some children.*

In this topic, students will use place value to estimate. Estimation strategies include rounding and using landmark and benchmark numbers to find "close to" sums and differences. Students often confuse rounding and estimating. The enVision glossary defines rounding as, "To replace a number with a number that tells about how much or how many to the nearest ten, hundred, thousand, and so on. Example: 42 rounded to the nearest 10 is 40" (p. G7). The enVision glossary defines estimating as, "To give an approximate number or answer" (p. G3).

Rounding is one type of estimation. Other computational estimation strategies include front-end methods and compatible numbers (Van de Walle, Karp, & Bay-Williams, 2016). To estimate the sum using rounding strategies 423 + 695 may round to 400 + 700. This estimate results in a sum of 1,100. Some children may round to 420 and 690 resulting in an underestimate. Some may even choose a compatible number strategy such as mentally combining 410 with the 690 the "make another 100" strategy. As long as students can justify their reasoning, accept 'close to' estimates.

Strategies for computational estimation should be chosen carefully. Rounding is not always the most accurate strategy for estimation because adding after rounding allows errors to accumulate. For example, I might estimate 444 + 649 as 400 + 600 = 1,000, despite the sum being closer to 1,100. The purpose of these strategies is to get children in the "ballpark" or in a range for the answer so they can reason, make sense and reflect when working on problems.

Third graders are expected work with place values to 10,000. In second grade, students "Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones…" (NVACS, 2010, 2.NBT.A.1), and "Read and write numbers to 1000 using base-ten numeral, number names, and expanded form" (NVACS, 2010, 2.NBT.A.3)". In fourth grade, students, "Generalize place value understanding for multi-digit whole numbers" (NVACS, 2010, 4.NBT.A.). Note that, "Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000" (NVACS, 2010, p. 29). This objective was added to bridge the gap from the 2nd to the 4th grade standards. Throughout this topic look for opportunities to review place value concepts from second grade and extend these to working with numbers up to ten thousand. Consider altering numbers in problems to provide these opportunities. Lessons 8-3, 8-6, and 8-7 focus on using place value understanding to round and estimate.

Focus Math Practice 4: Model with mathematics

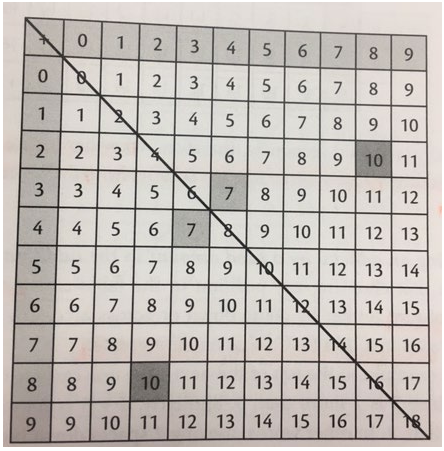
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs,

flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose (NVACS, 2010, p. 7).

To help students work towards security, encourage them to use multiple models and reflect on whether the results make sense. Problem solving lesson 8-9 is designed to reinforce this mathematical practice. See also the following resource: Reference Teacher's Edition (TE, pp. F24 - F24A).

Potential Misconception(s)

Students might confuse the addition and multiplication properties of operations for addition. For example, the Additive Identity Property of Zero (**5** + 0=**5**, ***a*** + 0=***a***) and the Multiplicative Identity Property of One (**5** x 1=**5** or ***a*** x 1= ***a***) are easily confused for students with limited conceptual understanding. See the *Properties of Operations Table* for additional information (NVACS, 2010, p. 90).



+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Students will benefit from testing these properties to build conceptual understanding. Is this rule always true? Can you find an exception to the rule? Consider creating a class developed anchor chart to collect examples of these properties.

For students struggling to understand the *Commutative Property of Addition*, consider extending lesson 8-2. Include work with the patterns in the addition table by using the table to show the symmetrical nature of the table. The included image is from the book *Uncomplicating Algebra* (Small, 2014) and shows the symmetry along the diagonal for 4 + 3 = 7 and 3 + 4 = 7, as well as, 3 + 7 = 10 and 7 + 3 = 10 (Commutative Property).

Looking Ahead:

On the Topic Performance Assessment, students will round numbers, estimate and justify their solutions to equations, and demonstrate a pattern on the addition table based on a context. Facilitate the development of problem-solving thinking habits. Provide opportunities for problem solving prior to working on the Topic Performance Assessment. When grading student work, accept responses that are mathematically reasonable even if they are not suggested in the Teacher's Edition.

Meaningful Fluency Practice & Assessment:

The following game can help students develop the mental math strategies that will support attaining NVACS standard 3.NBT.A.2, "Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction" (NVACS, 2010, p. 24). Consider rotating days so that students still have the opportunity to engage in meaningful practice of multiplication facts. There is a Pre/Post Addition/Subtraction Assessment available on the C &I, K-5 Mathematics website in resources for Academic Parent-Teacher Teams (APTT).

Rolling for 500: Estimation

See the directions for this game at the end of this document.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
Associative (Grouping) Property of Addition Commutative (Order) Property of Addition Identity (Zero) Property of Addition Compatible numbers	*round* *place value* *inverse operations* *multiples* *difference* *sum* *equation*

Additional terminology that students may need support with: *about, mental math, generalize, related*

***Collaborative Team Conversations (CTC)**
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding questions: "Are students developing conceptual understanding of the properties of addition in order to solve addition problems?"
"Are student estimating whole numbers based on reasoning?"

Lesson	Evidence	Look for
8-2	***Math Practices and Problem Solving*** (student work samples) Item 23	Focus CTC on the big idea: • student strategies and models. • student use of reasoning to analyze the relationship between addition and subtraction. • student use part/part/whole understanding when subtracting or adding.
8-3	***Quick Check*** (digital platform) Items 1, 4, and 5	Focus CTC on data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources". • rounding whole numbers based on reasonableness.

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 465-470	Use *Scoring Guide* TE pp. 465-470A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 8-1: Addition Properties		
3.NBT.A.2 MP.1 MP.3 **MP.4** MP.7 MP.8	**Access Prior Learning**: In Grade 2 students learned to add within 1,000 using models or strategies. **Securing the Big Idea:** Students are *securing* understanding of addition as problems that involve joining, part-part whole, or comparing. Students are also *securing* understanding of the Associative, Commutative, and Identity Properties of Addition.	**Topic Opener:** Introduce the *Topic Essential Question*, "How can sums and differences be estimated and found mentally?" (TE, p. 401). Consider making this an anchor chart in your classroom and allowing students to add strategies to the chart throughout the topic. You might also consider having students complete the *Review What You Know* prior to beginning instruction on Topic 8 so that you can respond to student instructional needs using the *Item Analysis for Diagnosis and Intervention* prior to beginning the topic (TE, p. 402-404). Consider introducing vocabulary as students encounter terminology in the lessons rather than introducing all terms at the beginning of the lesson. Add vocabulary to math word wall if possible. **Solve & Share:** Watch for students able to identify that there are the same number of buttons on both sides without having to compute. Pose questions to these students to evaluate whether they understand the Commutative Property of Addition. Watch for students that try to multiply the 3 addends. Facilitate discussion to identify that this is not a multiplicative situation. Is this repeated addition? Are we skip counting? After students share their solution methods and reasoning, consider asking students why this is an addition situation and not a multiplication situation (e.g. we are not joining *equal* groups to find a total). Consider discussing the *Look Back!* prompt so that the *Visual Learning Animation* can be used to confirm, clarify or correct student understanding. **Visual Learning:** After the Identity (Zero) Property of Addition has been introduced, consider comparing and contrasting the Additive Identity Property of 0 (Zero) and the Multiplicative Identity Property of 1 (versus the Multiplicative Zero Property), in order to confront common misconceptions regarding the properties. For more information on the differences read the *Instructional note* at the beginning of this topic. Consider discussing the *Convince Me!* to support students' development of MP. 4 "*Model with math.*" ***-continues on next page-***

		Assess and Differentiate: If time permits, teach students how to play the game *Tic Tac Toe* (TE, p. 409A). All students should have the opportunity to play this game. Consider the *Intervention Activity* for students who need additional support with the Commutative and Associative Properties (TE, p. 409A).
Lesson 8-2: Algebra-Addition Patterns		
3.OA.CD.9 MP.7 MP.8	**Access Prior Learning**: In Grade 2, students used the addition table to identify patterns in addition facts and work with even and odd numbers. In Grade 3, Topic 4, students explored patterns with even and odd numbers using a multiplication table. **Securing the Big Idea:** Students are *securing* their understanding of patterns in the addition table with even and odd numbers and properties of addition.	**Solve & Share:** In lesson 5-2, students compared and contrasted the addition and multiplication tables. Students have the opportunity to connect these ideas to foundations of mathematical argumentation, justification and proof. The *Solve & Share* enables students to play with numbers and identify patterns and rules that are not immediately apparent. This helps children understand what mathematics truly is and what it involves. **Visual Learning:** Consider having students use colored tiles to build the patterns during the video (01:27). If after viewing the *Visual Learning Animation* students still seem unsure about patterns for even and odd numbers with addition, consider viewing the *Another Look!* video. **Independent Practice/Math Practices and Problem Solving:** *Quick Check* item 3 asks students to recognize that the shaded numbers illustrate the Commutative Property of Addition. For more information on using the addition table to illustrate the addition properties read the *Instructional note* at the beginning of this topic. **Assess and Differentiate:** If time permits, you may consider replacing the *Problem-Solving Reading Mat* with the game *Tic Tac Toe* (TE, p. 409A) or the *Fluency Practice Activity* (TE, p. 459). Consider the *Intervention Activity* for students who need additional support with exploring the patterns when adding two even numbers, two odd numbers or an even and an odd number (TE, p. 415A).
Lesson 8-3: Round Whole Numbers		
3.NBT.A.1 MP.1 MP.3 MP.6	**Access Prior Learning**: In Grade 2, Topic 9, students learned how to compare 3-digit numbers using place value and a number line. **Developing the Big Idea:** Students *develop* understanding of place value to 10,000 and use place value to round using the language of "about" to signify approximation.	**Possible 2-day lesson to review place value to the 1,000 and introduce to the 10,000**. The ***Instructional note*** at the beginning of this topic describes the importance of these lessons. **Day 1:** **Solve & Share:** Consider building a class anchor chart to collect various strategies for finding approximations or 'close to' numbers. Students who struggle with place value understanding might benefit from the number line model. **Convince Me:** Consider assigning and discussing the *Convince Me!* to provide students the opportunity to reason with reasonable numbers based off of place value clues. After *Convince Me!* consider introducing place-value to the ten-thousands place by posting the place values and asking students what patterns they notice (e.g. the "ones" and "tens" from the one's period repeats into the thousands period). Pose numbers to have students practice rounding to the 1,000 and 10,000 place values. Ensure that students are justifying their rounding decisions rather than following a rule without understanding. **Day 2:** **Solve & Share:** Consider revisiting the original *Solve & Share* and ask students to solve again, with 1,728 stickers. **Visual Learning:** Consider watching the *Another Look!* video in place of the *Visual Learning Animation* and as a class round using the numbers 4,896 (e.g. rounding to hundreds is 4,900, round to thousands is 5,000) and 9,982 (e.g. rounding to hundreds is 10,000 rounding to thousands is 10,000). ***-continues on next page-***

		Strengthen students' number sense by discussing why when rounding 9,982 to both the hundreds and thousands place we end up 10,000. Consider providing time for students to use dice to roll 4 digit numbers, with a partner, that they practice rounding to both the hundred's and thousand's place values. **Assess and Differentiate:** If time permits, you may consider replacing the *Math and Science Activity* with the game *Tic Tac Toe* (TE, p. 409A) or the *Fluency Practice Activity* (TE, p. 459). Consider the *Intervention Activity* for students who need additional support with using a number line as a tool to round to 10 (TE, p. 421A). Consider utilizing the following question formats during practice: Example 1 Full Statement **Example Stem 1:** When rounding to the nearest ten, what is the **least** whole number that rounds to 50? Enter your answer in the response box. **Rubric:** (1 point) The student correctly enters the least/greatest whole number that rounds to the given number (e.g., 45). **Example Stem 2:** When rounding to the nearest ten, what is the **greatest** whole number that rounds to 50? Enter your answer in the response box. **Rubric:** (1 point) The student correctly enters the least/greatest whole number that rounds to the given number (e.g., 54). **Example Stem 3:** When rounding to the nearest hundred, what is the **least** whole number that rounds to 500? Enter your answer in the response box. **Rubric:** (1 point) The student correctly enters the least/greatest whole number that rounds to the given number (e.g., 450). **Example Stem 4:** When rounding to the nearest hundred, what is the **greatest** whole number that rounds to 500? Enter your answer in the response box. **Rubric:** (1 point) The student correctly enters the least/greatest whole number that rounds to the given number (e.g., 549). **Example Stem 5:** When rounding to the nearest ten, what is the **least** whole number that rounds to 520? Enter your answer in the response box. **Rubric:** (1 point) The student correctly enters the least/greatest whole number that rounds to the given number (e.g., 515). **Example Stem 6:** When rounding to the nearest ten, what is the **greatest** whole number that rounds to 520? Enter your answer in the response box. **Rubric:** (1 point) The student correctly enters the least/greatest whole number that rounds to the given number (e.g., 524).
Lesson 8-4: Mental Math-Addition		
3.NBT.A.2 MP.1 MP.3 **MP.4** MP.5 MP.6 MP.7	**Access Prior Learning**: In Grade 2, students learned to decompose and then recomposing numbers into 'friendly numbers' as a strategy for simplifying an addition problem and finding the solution mentally. **Developing the Big Idea:** Students further *develop* an understanding that there is more than one strategy for working with mental math, including using place value understandings.	**Solve & Share:** To encourage students to use mental math strategies, consider removing writing tools, instead, have students describe their solution strategy while their partner represents this thinking in their book, a whiteboard or in math journals. As students share their strategies and reasoning, consider charting the different mental math strategies students create. Highlight a few strategies that are the most efficient given the numbers in the problem. **Visual Learning:** If the strategies in the *Visual Learning Animation* are not already on the chart created, add these mental math methods to the poster. If they are already there, explicitly connect those in the *Visual Learning Animation* to the student's strategy. Ask students, "How using friendly numbers (multiplies of 10), makes solving problems easier?" Consider assigning the *Convince Me!* to give students the opportunity to practice the methods introduced in the *Visual Learning Animation* and support students' development of MP. 4, *Model with Mathematics.* ***-continues on next page-***

		Assess and Differentiate: If time permits, teach students how to play "Clip and Cover" (TE, p. 427A). All students should have the opportunity to play this game. Consider the *Intervention Activity* for students who need additional support with using place value parts to add (TE, p. 427A). Connect this work to students using expanded notation to connect partial sums.
Lesson 8-5: Mental Math-Subtraction		
3.NBT.A.2 MP.1 MP.3 **MP.4** MP.7 MP.8	**Access Prior Learning**: In 2nd grade students worked with compensation as a strategy. In the previous lesson, students developed understanding of mental math strategies for addition with decomposition and compensation. **Developing the Big Idea:** Students further *develop* an understanding that there is more than one way to do mental math including compensation with subtraction.	**Instructional Note:** To help students understand how compensation works with subtraction, consider investigating constant difference using the number line model. For example, the difference between two individuals' birth years will be the same as the difference between these two individuals' ages. Also, the "hop" on a number line can be slid up or down a number line to a benchmark number. **Solve & Share:** To encourage students to use mental math strategies, consider removing writing tools, instead, students to describe their solution strategy while their partner represents this thinking in their book, mathematics journal or on a whiteboard. As students share their strategies and reasoning, consider making a poster of the different mental math strategies students use. **Visual Learning:** Add the mental math methods introduced in the *Visual Learning Animation* to the poster of mental math strategies. Ask students, "How does using place value to solve subtraction problems make mental math easier?" **Assess and Differentiate:** If time permits, you may consider replacing the *Problem Solving Reading Mat* with the game *Tic Tac Toe* (TE, p. 409A), *Clip and Cover* (TE, p. 427A), or the *Fluency Practice Activity* (TE, p. 459). Instead of using the *Intervention Activity* for students whom need additional support, work with the idea of 'constant difference' and modeling this first on a number line and later applying this idea to compensation.
Lesson 8-6: Estimate Sums		
3.NBT.A.2 MP.2 MP.3 **MP.4** MP.6	**Access Prior Learning**: In 2nd grade, students worked with compatible numbers. In previous lessons in this topic, students have worked with rounding numbers and developed mental math strategies for addition. **Developing the Big Idea:** This lesson *develops* students' understanding of the purpose of estimation and strategies for estimating with addition using compatible numbers.	**Solve & Share:** After the *Solve & Share* ask students what estimation strategies they used (e.g. rounding is one strategy for estimation). Ask, "How does estimating sums help us?" Watch for students that solve for an exact number and support by asking the *Ask Guiding Questions as Needed* (TE, p. 435) prompts. Consider providing number lines to support conceptual understanding (Teaching Tool 7). Have students share their reasoning with the class. Students benefit from both successful strategies and the discussion of common misconceptions. **Convince Me:** Consider having students partner talk the *Convince Me!*. Discuss as a whole group as this supports students development of using number sense to estimate. **Assess and Differentiate:** If time permits, teach students how to play *Clip and Cover* (TE, p. 439A). All students should have the opportunity to play this game. Consider the *Intervention Activity* for students who need additional support with using estimation strategies such as rounding or compatible numbers (TE, p. 439A). ***-continues on next page-***

Consider utilizing the following question format during practice:

Example 1
Full Statement

Example Stem 1: Select if each sum is greater than 80 or less than 80.

	Greater than 80	Less than 80
41 + 42		
33 + 35		
41 + 36		
46 + 37		

Rubric: (1 point) The student enters the correct value for the unknown (e.g., GLLG).

Example 4
Full Statement

Example Stem 4: Identify if each sum is closer to 70 or closer to 80.

	Closer to 70	Closer to 80
32 + 47		
26 + 51		
35 + 37		

Rubric: (1 point) The student enters the correct value for the unknown (e.g., 80, 80, 70).

Example Stem 5: Select the table to show if each sum is closer to 400 or closer to 500.

	Closer to 400	Closer to 500
302 + 105		
398 + 49		
212 + 247		
196 + 251		

Rubric: (1 point) The student enters the correct value for the unknown (e.g., 400, 400, 500, 400).

Lesson 8-7: Estimate Differences

3.NBT.A.2

MP.1
MP.2
MP.4
MP.8

Access Prior Learning:
In previous lessons in this topic, students have worked with rounding numbers and developed mental math strategies for subtraction.

Developing the Big Idea:
This lesson *develops* students' understanding of the purpose of estimation and strategies for estimating with subtraction.

Solve & Share:
Watch for students that solve for an exact number and support by asking the *Ask Guiding Questions as Needed* (TE, p. 441) prompts. Consider also supporting students struggling to estimate by providing them with number lines (Teaching Tool 7).

Consider discussing the *Look Back!* prompt to support students' development of estimation strategies.

Visual Learning:
After viewing the *Visual Learning Animation*, consider asking students if there is another way to estimate the difference. This is an opportunity to build understanding that there are multiple ways to estimate (e.g. rounding, front-end, compatible numbers). Students might round 493 to the hundreds place because it is so close to 500, while they could round 126 to the nearest tens place. Rounding the numbers to different place values to make an easier estimate applies to *Quick Check* item 22.

Independent Practice/Math Practices and Problem Solving:
Quick Check item 22 aligns to *Visual Learning Animation*. An emphasis of this lesson has been on estimates giving different answers. Therefore, remember to accept student work that is mathematically reasonable.

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		Assess and Differentiate: If time permits, teach students how to play *Teamwork* (TE, p. 445A). All students should have the opportunity to play this game. Consider the *Intervention Activity* for students who need additional support with estimating differences using rounding (TE, p. 455A). Consider utilizing the following question format during practice: **Example Stem 2:** Select if each difference is greater than 40 or less than 40. **Rubric:** (1 point) The student enters the correct value for the unknown (e.g., GGLL).

	Greater than 40	Less than 40
83 – 40		
85 – 43		
83 – 45		
80 – 43		

Lesson 8-8: Relate Addition and Subtraction		
3.NBT.A.2 MP.2 MP.3 **MP.4**	**Access Prior Learning**: In Grade 2, students developed understanding of addition and subtraction as inverse operations. In previous lessons, students have solved addition and subtraction problems using mental math and estimation strategies. In Topic 4, students learned about the inverse relationship between multiplication and division. **Developing the Big Idea:** This lesson *develops the* understanding of addition and subtraction as inverse operations.	**Solve & Share:** Watch for students who select numbers and operations without reasoning. For these students return to the context of the problem, focus on understanding the question and developing a problem solving plan, rather than focusing on a solution and consider replacing the numbers with the word "some" to help the students understand the context. Ask students that finish first, how they could prove their solutions are correct. Consider having students that are able to successfully check share their strategy so that as a class you can discuss why we can check a subtraction problem with addition (e.g. their inverse relationship). **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 10 for distributed practice of telling time using an analog clock. **Assess and Differentiate:** If time permits, you may consider replacing the *Problem Solving Reading Mat* with the game *Tic Tac Toe* (TE, p. 409A), *Clip and Cover* (TE, p. 427A; 439A), *Teamwork* (TE, p. 445A), or the *Fluency Practice Activity* (TE, p. 459). Consider the *Intervention Activity* for students who need additional practice using the inverse operation to check answers (TE, p.277A).
Lesson 8-9: Math Practices and Problem Solving- Model with Math		
3.NBT.A.2 **MP.4** MP.1 MP.2 MP.3 MP.5	**Access Prior Learning:** In Topic 2 students focused on MP.4 to solve multi-step problems. Throughout Topic 8 students have modeled the math with number lines, bar diagrams, and equations. **Developing the Big Idea:** In this lesson, students further *develop* their understanding of MP. 4 *"Model with Mathematics"* by using bar diagrams and equations to represent problems that are more complex.	This lesson provides an opportunity to focus on the thinking habits and display the behaviors associated with Math Practice 4, *"Model with math*." Refer to the *Math Practices and Problem Solving Handbook* (TE, p. F24-F24A, F29) for suggestions on how to develop, connect and assess this Math Practice. Also, reference the handbook in the Student Edition (SE, p. F24). **Solve & Share:** Consider reintroducing MP. 4 (SE p. F24) before introducing the *Solve & Share*. After introducing, the *Solve & Share* identify what the problem is asking (e.g., how many total minnows are in the pond?). Consider asking how this problem is similar to the last *Solve & Share* (e.g. comparing quantities). Consider then asking students how they could model this problem (e.g. bar diagrams, number line, equations). You might also consider using the time where students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.4 that are listed in the *Math Practices and Problem Solving Handbook* (TE p. 24A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice. **Convince Me:** Consider assigning the *Convince Me!* as it provides an opportunity for students to reason when it is appropriate to use a bar diagram. ***-continues on next page-***

		Assess and Differentiate: If time permits, teach students how to play *Teamwork* (TE, p. 457A) as this relates the mathematics developed in this topic to a real world context. Consider the *Intervention Activity* for students who need additional support with adding using a bar diagram (TE, p. 457A).

References

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Common Core State Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). K-5, Numbers in Operations Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Small, M. (2014). *Uncomplicating algebra to meet common core standards in math, K-8.* New York: Teachers College Press, Nelson Education.

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Rolling for 500

Grade Level: 3-5

Number of Players: 2-4

Materials Needed:
- a die
- a game piece for each player
- game board

Mathematical Understanding:
Students strengthen numerical fluency through practice with strategies used for addition and subtraction.

Object of the Game:
The first player to reach or cross the **Finish** wins the game.

Directions:
Each player places their marker on the **Start** square of the shared game board.

Player 1 rolls the die. Match the number rolled to the table on the game board to determine how many spaces to move forward or backward. Player 1 moves their marker.

Players take turns rolling the die and using the table to determine spaces moved.

The first player to reach or cross the **Finish** line wins the game.

Players cannot move below zero and wait at the start space for a positive roll.

Two players can be on the same space on the gameboard at the same time.

Optional:
When playing the estimation version, players can state out loud what their exact space would be and how close they are to the space they move onto to. Which space is the closest and why?

Guiding Questions:
What do you know?
Where do you think you will begin?
Where are you stuck? What is confusing? What are you wondering about?
What are you going to try?
What did you think about to come to your answer?

Differentiation:

Two versions of the game can be used for grades 3-5. **Rolling for 500** gives practice with place value strategies to add and subtract numbers up to 500. **Rolling for 500 estimation** gives practice with place value strategies for addition and subtraction and also requires comparative reasoning in order to properly place the gameboard marker.

Game Trajectory:
Pre K-K: Counting along a number line to 20

K-2: Addition and subtraction to get to 50

3-5: Rolling for 500 or Rolling for 500 estimation version

5-6: Rolling for 5

Clean up Checklist for Game Bag:
Die
Game piece markers
Game boards

Rolling for 500

Roll	Spaces
1	add 30
2	subtract 20
3	add 50
4	subtract 60
5	add 80
6	add 10

Start →	13	18	33	38	49	62	74	80	91	103
	111	123	136	145	150	168	172	181	199	205
	208	220	231	244	256	265	277	284	291	302
	310	319	335	342	358	369	372	385	392	400
	413	420	434	446	450	454	467	472	488	500 Finish

Rolling for 500 Estimation Game Board

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Grade 3 Topic 9: Fluently Add and Subtract Within 1,000

Topic 9
Fluently Add and Subtract Within 1,000
Number of Lessons: **8**
F/D/E: **4** days
NVACS Focus: NBT.A
Total Days:~12

[3rd Grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: Numbers and Operations in Base Ten (p. 12)

Prior to instruction, view the Topic 9 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 401A-401F), the Topic Planner (pp.47IA-471C), all 8 lessons, and the Topic Assessments (pp. 533-534A).

Mathematical Background: Read Topic 9 Cluster Overview/Math Background (pp. 401A-401F)	**Topic Essential Question:** What are standard procedures for adding and subtracting whole numbers? *Reference Answering the Topic Essential Question (TE, pp. 529-530) for key elements of answers to the Essential Question.*

***The lesson map for this topic is as follows*:**

9-1	9-2	9-3	9-4	9-5	9-6	9-7	9-8	Assessment

4 F/D/E days used strategically throughout the topic.

Instructional note:

As previously stated, this topic is part of a cluster that includes topics 8 and 10. These topics focus on using place-value understanding and properties of operations to perform multi-digit arithmetic. A big idea specific to topic 9 is the use of strategies and algorithms based on place-value understanding for solving multi-digit addition and subtraction problems within 1,000 (3.NBT.A.2). This understanding builds on the work done in second grade with standard 2.NBT.B.7 that states, "Add and subtract within 1,000, using concrete models or drawing and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; **relate the strategy to a written method**. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds" (p.19).

The phrase "relate the strategy to a written method" is bolded to point out that third graders should have had experiences with using a strategy to add and subtract within 1,000 and relating that to a written method. Fuson and Beckmann (2012) explain a trajectory from strategies, to written method, to standard algorithm (the full article can be accessed in the references section at the end of this document). See figures below (p. 19 and 21).

FIGURE 2: *Multidigit Addition Methods that Decompose into Base Ten Units*

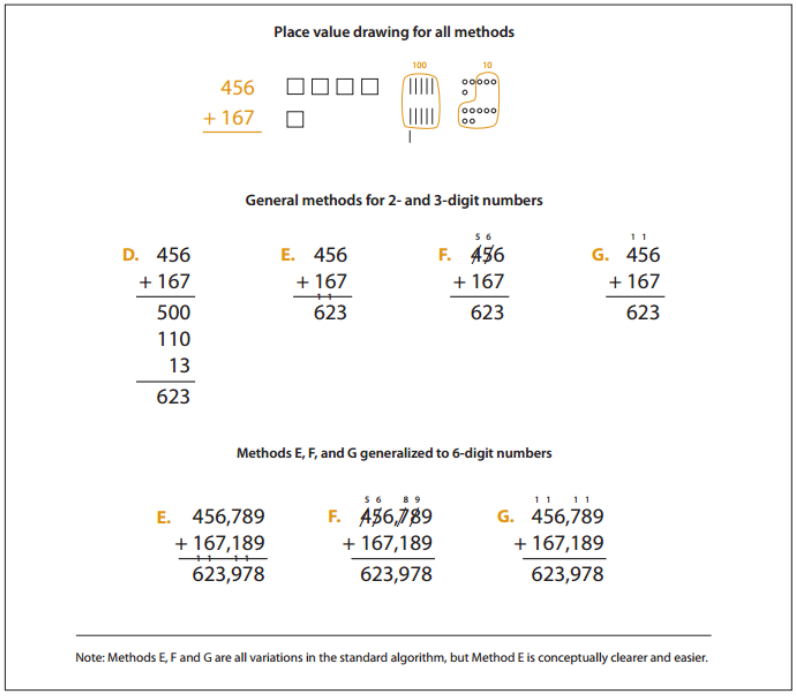
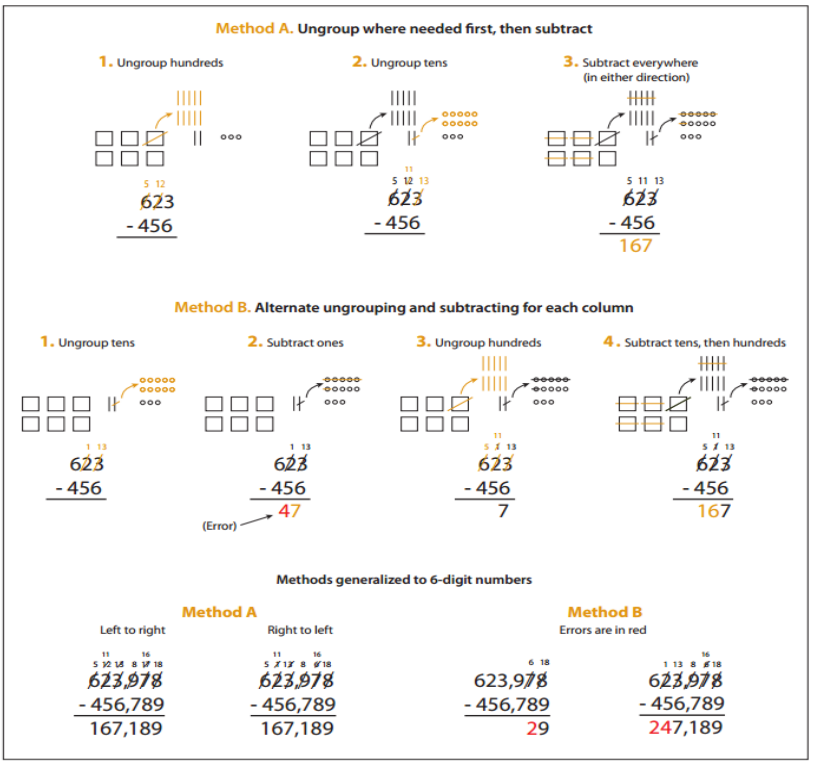


FIGURE 3: *Multidigit Subtraction Methods that Decompose into Base Ten Units*



These ideas relate to topic 9 as students move through various models and strategies towards a standard algorithm.

For example, in the problem 175 + 366 students might say, "I carried the 1 ten in 11 to the tens place to add with the other tens. Then I added 10 +70 + 60 to make 140 so I wrote the 4 under the tens place and carried the 1 hundred to the hundreds place…"

Students might also explain that when adding the 1 (10), 7 (70), and 6 (60) in the tens place they had 14 tens and since 10 tens is 100 they carried a 1 to be added with the rest of the hundreds.

The essential question for this topic is, *"What are standard procedures for adding and subtracting whole numbers?"* This connects to the learning trajectory for the topic which **focuses on connecting partial sums and differences to the expanded algorithm and then to the standard algorithm**. In 4th grade, students are expected to demonstrate security with a standard algorithm. The goal of this unit is to secure the language of regrouping and the concepts of partial sums and differences.

Focus Math Practice 3: Construct viable arguments and critique the reasoning of others
The standard states, "They (students) make conjectures and build a logical progression of statements to explore the truth of their conjectures…. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and -if there is a flaw in and argument- explain what it is" (NVACS, 2010, p. 6-7).

To help students work towards security, consider introducing a routine that when students are done working on the *Solve & Share* they explain their reasoning to themselves to "convince yourself." After practicing, students could share their reasoning and conjectures with a classmate to "convince a friend." Finally, they share their reasoning with somebody that disagrees with their conjecture to "Convince a critic." Consider also requiring that if students feel there is flawed reasoning they must be able to explain the flaw or when a student revises their own thinking they must be able to explain why they have changed their conjecture. Behaviors associated with MP.3 are described in the Teacher's Edition (pp. F23 - F23A) and the NVACS.

Meaningful Fluency Practice & Assessment:
The following games can help students develop strategies that will support attaining NVACS standard 3.NBT.A.2, "Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction (NVACS, 2010, p. 24)." Consider rotating days so that students still have the opportunity to engage in meaningful practice of multiplication facts. There is a Pre/Post Addition/Subtraction Assessment available on the C & I, K-5 Mathematics website in resources for Academic Parent-Teacher Teams (APTT).

Rolling for 500

Directions: See below in this document for the directions and game board.

Close Call: Addition Version

Directions**:** Remove 10s and face cards from the deck. Shuffle the deck and deal each player 8 cards. Each player selects six of their cards and creates two 3-digit numbers from them. The goal is to create two numbers that have a sum as close to 1000 as possible, without going over. After players have made their selections, they place their cards face up in front of them, arranging them so other players can see which two numbers they have created. The player with the sum closest to 1000, without going over, wins a point. In the case of a tie, a point is awarded to each team. Shuffle the cards before dealing another round. Play continues for 5 rounds. The player with the most points after the last round wins the game.

For example, a student draws 7, 4, 5, 6, 8, 2, 1, 1 and chooses to use the cards 7, 4, 5, 6, 8, 1, creating the problem 754 +186 = 940. Just so long as another player does not get a number closer to 1000 without going over then this student earns 1 point for this round.

Close Call: Subtraction Version

Directions**:** Remove 10s and face cards from the deck. Shuffle the deck and deal each player 8 cards. Each player selects five or six of their cards and creates two 3-digit numbers or a 2-digit number that's subtracted from a 3-digit number from the dealt cards. The goal is to create two numbers that have a difference as close to 10 as possible, without going lower. After players have made their selections, they place their cards face up in front of them, arranging them so other players can see which two numbers they have created. The player with the numbers closest to 10, without going lower, wins a point. In the case of a tie, a point is awarded to each team. Shuffle the cards before dealing another round. Play continues for 5 rounds. The player with the most points after the last round wins the game. For example, a student draws 7, 4, 5, 6, 8, 2, 1, 1 and chooses to use the cards 7, 4, 5, 6, 8, 1, creating the problem 821 - 765 = 56. Just so long as another player doesn't get a difference closer to 10 without going lower then this student earns 1 point for this round.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
conjecture	*regroup* *compatible numbers* *Associative Property of Addition* *Commutative Property of Addition* *inverse operation* *expanded form*

Additional terminology that students may need support with: *more, fewer, less*

***Collaborative Team Conversations (CTC)**

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students applying place value understanding to add and subtract whole numbers?"

Lesson	Evidence	Look for
9-1	***Quick Check*** (digital platform) Items 1, 3, and 4	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources". • understanding place value to break apart (decompose) numbers.
9-5	***Math Practices and Problem Solving*** (student work samples) Item 16	Focus CTC around the big idea: • student strategies and models. • students understanding place value to break apart numbers. • students using partial differences.

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 529-534	Use *Scoring Guide* TE pp. 529-534A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 9-1: Use Partial Sums to Add		
3.NBT.A.2 MP.1 **MP.3** MP.4 MP.5 MP.7	**Access Prior Learning:** In Topics 4 & 6, Grade 2 students developed fluency with adding and subtracting 2-digit numbers. In Topics 10 & 11, Grade 2 students began to develop understanding for adding and subtracting 3-digit numbers using models such as place value blocks and number lines. In Topic 8, students revisited the properties of addition and used the number line to model addition and subtraction. **Developing the Big Idea:** Students are further *developing* their understanding of using place value strategies to add 3-digit numbers by using the expanded algorithm to break the addition problems into a series of easier problems based on place value, then partial sums are added together to get the total.	**Topic Opener:** Introduce the ***Topic Essential Question***, "What are standard procedures for adding and subtracting whole numbers?" (TE, p. 471). Consider making this an anchor chart in your classroom. Adding new ideas to the chart helps students to see the development of concepts and strategies and make new connections. You might also consider having students complete the *Review What You Know* prior to beginning instruction on topic 9 so that you can respond to student instructional needs using the *Item Analysis for Diagnosis and Intervention* (TE, pp. 472-474). Consider introducing vocabulary terms as they are encountered in the lessons rather than introducing all terms at the beginning of the lesson. Consider introducing the Convince Yourself, Convince a Friend, and Convince a Critic at this time to support student development of MP. 3 "*Construct viable arguments and critique the reasoning of others."* **Solve & Share:** Consider having a place value chart (Teaching Tool 5) and base-10 blocks available, especially for students that struggled with using place value in topic 8. Watch for students that are attempting to or do use the U.S. Traditional Algorithm. For these students ask questions to determine if the student has conceptual understanding of the procedures or if it is just a memorized procedure. Encourage students that only have a memorized procedure to show their work using the place value chart and base-10 blocks or to attempt a partial sums strategy such as the expanded algorithm and explain their reasoning. Watch for students that do not line up digits by place value. Consider posing a situation where the digits are not lined up correctly and discussing the error with the whole class. Can the students use this error to generalize and make a rule about place value and addition? Consider beginning to support students' development of the meaning of the mathematical term "regroup" by connecting their informal language for the term with the actual term. **Assess and Differentiate:** If time permits, you may consider replacing the *Math and Science Activity* with games from previous topics or the *Fluency Practice Activity* (TE, p. 523). Child-watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 479A). ***CTC: *Quick Check*** (digital platform) Consider utilizing the following question format during practice: **Example Stem 1:** What unknown number makes this equation true? 763 + 97 = 763 + 100 – □ Enter your answer in the response box. **Rubric:** The student enters the correct number to make the equation true (e.g., 3). **Example Stem 2:** What unknown number makes this equation true? 763 + 104 = 763 + 100 + □ Enter your answer in the response box. **Rubric:** The student enters the correct number to make the equation true (e.g., 4). ***-continues on next page-***

		Example Stem 1: What unknown number makes this equation true? 763 + 7 = 700 + □ Enter your answer in the response box. **Rubric:** The student enters the correct number to make the equation true (e.g., 70). **Example Stem 2:** What unknown number makes this equation true? 763 + 43 = 800 + □ Enter your answer in the response box. **Rubric:** The student enters the correct number to make the equation true (e.g., 6).
Lesson 9-2: Add 3-Digit Numbers		
3.NBT.A.2 MP.1 **MP.3** MP.4 MP.5	**Access Prior Learning:** In the previous lesson, students used the expanded algorithm for adding 3-digit numbers. **Developing the Big Idea:** Students are *developing* the understanding of a standard algorithm as a shortcut for the expanded algorithm by connecting it to place value understanding.	**Solve & Share:** Consider having place value charts (Teaching Tool 5) and base-10 blocks available for students still needing to direct model the addition and build place value understanding. Watch for students that only use the U.S. Traditional Algorithm. Encourage them to think about whether that is always the most appropriate strategy. Students build fluency when they think flexibly and determine that different strategies can be more efficient because of the numbers in a problem. This type of reasoning builds number sense and place value understanding. Consider supporting students' development of the meaning of the mathematical term "regroup" by connecting their informal language for the term with the actual term. If students did not offer a solution method similar to "KiKo's Work", consider discussing "KiKo's Work" as a class (TE, p. 481). Notice that Kiko offers a visual representation to explain the regrouping he did in the standard algorithm. **Look Back:** Consider discussing the *Look Back!* prompt to continue to support students' understanding of estimation and identify the connections in mathematical ideas. **Visual Learning:** The *Visual Learning Animation* only shows the U.S. Traditional Algorithm as a solution method. Research has shown that once students have been taught a standard algorithm, they are unlikely to go back to using invented algorithms and reasoning strategies. If students are insecure in their place value understanding, consider not showing the *Visual Learning Animation* and instead run item 19 from *Math Practices and Problem Solving* as another *Solve & Share* to give more opportunity to work with invented algorithms and connect them to place value understanding. **Convince Me:** If you do show the *Visual Learning Animation*, consider assigning and discussing the *Convince Me!* to support students' development of understanding the math behind procedures such as the U.S. Traditional Algorithm. **Independent Practice/Math Practices and Problem Solving:** *Quick Check* item 22 *Higher Order Thinking* may be more difficult for students because so far this year students have mostly worked with Add to-Result Unknown problem types (for more information on problem types see p. 88 of NVACS). Item 22 is a "Compare-Bigger Unknown" problem type. **Assess and Differentiate:** If time permits, teach students how to play the game "Clip and Cover" (TE, p. 485A). All students should have the opportunity to play this game. Child watch to identify students who need additional support and consider the included *Intervention Activity* (TE, p. 485A). ***-continues on next page-***

		Consider utilizing the following question formats during practice: **Example Stem 1:** What unknown number makes this equation true? 763 + 7 = 700 + □ Enter your answer in the response box. **Rubric:** The student enters the correct number to make the equation true (e.g., 70). **Example Stem 2:** What unknown number makes this equation true? 763 + 43 = 800 + □ Enter your answer in the response box. **Rubric:** The student enters the correct number to make the equation true (e.g., 6).
Lesson 9-3: Continue to Add 3-Digit Numbers		
3.NBT.A.2 MP.1 MP.2 **MP.3** MP.6 MP.7 MP.8	**Access Prior Learning:** In the previous lesson, students learned to add 3-digit numbers using the U.S. Traditional Algorithm and place value understanding. **Developing the Big Idea:** Students further *develop* the understanding of the U.S. Traditional Algorithm as a shortcut for the expanded algorithm by connecting it to place value understanding.	**Instructional Note:** Many problems in this lesson are the compare problem type, which can be difficult for learners still building an understanding of addition. Watch for students that seem to be struggling due to the comparative terminology and provide vocabulary support for the terms as well as questioning to push students to think about what is known and unknown. Ask them to connect back to the situation given and the relationship between these terms. Focus on making sense of problems and relationships helps students develop problem-solving habits. **Solve & Share:** Prior to introducing the *Solve & Share*, consider asking students to share out all the strategies they can think of for adding multi-digit numbers and posting these ideas to the anchor chart. Keep in mind that the outcome of this topic is not that students are fluent with the U.S. Traditional Algorithm, but rather that they are secure with the place value understandings to add multi-digit numbers. Therefore, it is acceptable for students to be using strategies based on place value understanding such as the expanded or partial sums algorithms to solve multi-digit addition problems. **Visual Learning:** The *Visual Learning Animation* only shows the U.S. Traditional Algorithm as a solution method. Consider pausing frequently to connect the computation used in the U.S. Traditional Algorithm to partial sums. Include modeling with using base-10 blocks if needed. You might also consider not showing the *Visual Learning Animation*, but still ask students to share their invented algorithms, or partial sums, and explain using place value understanding. **Assess and Differentiate:** If time permits, teach students how to play the game "Teamwork" (TE, p. 491A). All students should have the opportunity to play this game. Child watch to identify students who need additional support and consider the included *Intervention Activity* (TE, p. 485A and/or TE, p. 491A). Consider utilizing the following question formats during practice: **Example Stem 1:** What unknown number makes this equation true? 763 + 29 = □ Enter your answer in the response box. **Example Stem 2:** What unknown number makes this equation true? □ = 763 + 29 Enter your answer in the response box. **Rubric:** The student enters the correct difference (e.g., 792).

Lesson 9-4: Add 3 or More Numbers

3.NBT.A.2 MP.2 **MP.3** MP.4 MP.8	**Access Prior Learning:** In the previous lesson students learned to add two, 3-digit numbers using a standard algorithm, like the U.S. Traditional Algorithm. **Developing the Big Idea:** Students further *develop* an understanding of the U.S. Traditional Algorithm as a shortcut for the expanded algorithm by connecting it to place value understanding with 3 multi-digit numbers.	**Solve & Share:** After introducing the *Solve & Share*, consider asking students to make sense of the problem, identify what the problem is asking and develop a plan. To continue to support students' understanding of additive versus multiplication situations consider also asking students why this is not a multiplication problem (e.g. we do not have repeated equal sized groups of fish). **Visual Learning:** The *Visual Learning Animation* only shows the U.S. Traditional Algorithm as a solution method. Consider pausing frequently to connect the computation of the U.S. Traditional Algorithm to partial sums. Include modeling with using base-10 blocks if needed. You might also consider not showing the *Visual Learning Animation*, but still have students solve and share their invented algorithms, or partial sums, with place value understanding. **Independent Practice/Math Practices and Problem Solving:** Consider using item 18 as it has students use information from a picture to find the total height. **Assess and Differentiate:** If time permits, teach students how to play "Display the Digits" (TE, p. 497A). All students should have the opportunity to play this game. Child watch to identify students who need additional support and consider the included *Intervention Activity* (TE, p. 497A). Consider utilizing the following question formats during practice: **Example Stem 1:** What unknown number makes this equation true? 763 + 97 = 763 + 100 – □ Enter your answer in the response box. **Rubric:** The student enters the correct number to make the equation true (e.g., 3). **Example Stem 2:** What unknown number makes this equation true? 763 + 104 = 763 + 100 + □ Enter your answer in the response box. **Rubric:** The student enters the correct number to make the equation true (e.g., 4).

Lesson 9-5: Use Partial Differences to Subtract

3.NBT.A.2 **MP.3** MP.6 MP.7 MP.8	**Access Prior Learning:** In topics 4 & 6, Grade 2 students developed fluency with adding and subtracting 2-digit numbers. In topics 10 & 11, Grade 2 students began to develop understanding for adding and subtracting 3-digit numbers using models such as place value blocks and number lines. In topic 8, students revisited the properties of addition, compensation with subtraction, and using the number line to model addition and subtraction. **Developing the Big Idea:** Students are further *developing* their understanding of using place value strategies to subtract 3-digit numbers by using the expanded algorithm to break the subtraction problems into a series of easier problems based on place value.	**Solve & Share:** Consider having a place value chart (Teaching Tool 5) and base-10 blocks available, especially for students that are struggling to connect regrouping in addition to place value understanding. Watch for students that attempt or do use the U.S. Traditional Algorithm. For these students ask questions to determine if the student has conceptual understanding or if they are using a memorized procedure. Are they able apply place value understanding to use a partial differences strategy? Are they making simpler problems and showing flexibility and efficiency in their thinking? Encourage all students to show or explain their reasoning and to connect these ideas to a written strategy. For an example of how concrete modeling can be connected to a representational strategy, see *Analyze Student Work* in Lesson 9-6 (TE, p. 505); "Ira's Work". **Visual Learning:** The *Visual Learning Animation* only shows the U.S. Traditional Algorithm as a solution method, research states that once students have been taught a standard algorithm they are unlikely to go back to using invented algorithms. Consider pausing frequently to connect the computation with the U.S. Traditional Algorithm to the expanded algorithm and partial differences; include using base-10 blocks if needed. You might also consider not showing the *Visual Learning Animation* and instead ask students to share their strategies and reasoning for solving. Are students applying addition strategies to help them subtract? Are there similarities in the use of place value between strategies that students can notice and use to make a generalization about place value and subtraction? ***-continues on next page-***

		Convince Me: Consider assigning and discussing the *Convince Me!* to support students' development of using place value understanding to subtract multi-digit numbers. **Independent Practice/Math Practices and Problem Solving:** Item 15 *Higher Order Thinking* of the *Quick Check* is a "Compare" problem which may present an extra challenge. Students have been working mostly with "Take from-Result Unknown" problems in this lesson (for more on the problem types see page 88 of the NVACS). Focus on helping students make sense of what problems are asking and developing a plan. **Assess & Differentiate:** If time permits, you may consider replacing the *Problem Solving Reading Mat* with the game "Clip and Cover" (TE, p. 485A), "Teamwork" (TE, p. 491A), "Display the Digits" (TE, p. 497A), or the *Fluency Practice Activity* (TE, p. 523). Child watch to identify students who need additional support and consider the included *Intervention Activity* (TE, p. 503A). ***CTC: *Math Practices and Problem Solving*** (student work samples)
Lesson 9-6: Subtract 3-Digit Numbers		
3.NBT.A.2 MP.1 MP.2 MP.4 MP.5 MP.8	**Access Prior Learning:** In previous lessons in this topic students subtracted multi-digit numbers by breaking larger subtraction problems into smaller problems. Students found partial differences that helped them find differences for larger subtraction problems. By using the expanded algorithm, students further developed their understanding of place value. **Developing the Big Idea:** This lesson further *develops* students' understanding of using place value strategies to subtract 3-digit numbers by using the expanded algorithm to break the subtraction problems into a series of easier problems based on place value. Students also *begin* to understand use of the standard algorithm for subtracting 3-digit numbers.	**Solve & Share:** Consider having place value charts (Teaching Tool 5) and base-10 blocks available, especially for students still needing to direct model the subtraction with regrouping. The *Solve & Share* has extraneous information (e.g. the number of houses for sale in Hunter County); consider asking students the questions provided in the *Build Understanding* (TE, p. 505) to make sure all students are working within the same constraints of the problem. **Visual Learning:** The *Visual Learning Animation* only shows the U.S. Traditional Algorithm as a solution method, research states that once students have been taught a standard algorithm they are unlikely to go back to using invented algorithms. Consider pausing frequently to connect the computation of the standard algorithm to the expanded algorithm or a partial differences strategy, include using base-10 blocks if needed. You might also consider not showing the *Visual Learning Animation*, but still have students solve and share their invented algorithms, or expanded algorithms, with place value understanding. **Convince Me:** If you do show the *Visual Learning Animation*, consider assigning and discussing the *Convince Me!* to support students' development of understanding the math behind the procedures in the U.S. Traditional Algorithm. **Assess & Differentiate:** If time permits, you may consider replacing the *Math and Science Activity* with the game "Clip and Cover" (TE, p. 485A), "Teamwork" (TE, p. 491A), "Display the Digits" (TE, p. 497A), or the Fluency Practice Activity (TE, p. 523). Child watch to identify students who need additional support and consider the included *Intervention Activity* (TE, p. 509A). Consider utilizing the following question formats during practice: **Example Stem 2:** What unknown number makes this equation true? 763 – 104 = 763 – 100 – □ Enter your answer in the response box. **Rubric:** The student enters the correct number to make the equation true (e.g., 4).

		Example Stem 1: What unknown number makes this equation true? 763 – 43 = 763 – 40 – □ Enter your answer in the response box. **Rubric:** The student enters the correct number to make the equation true (e.g., 3). **Example Stem 1:** What unknown number makes this equation true? 763 – 96 = □ Enter your answer in the response box. **Rubric:** The student enters the correct difference (e.g., 667). **Example Stem 2:** What unknown number makes this equation true? □ = 763 – 96 Enter your answer in the response box. **Rubric:** The student enters the correct difference (e.g., 667).
Lesson 9-7: Continue to Subtract 3-Digit Numbers		
3.NBT.A.2 MP.1 MP.2 **MP.3** MP.4 MP.5 MP.8	**Access Prior Learning:** In the previous lesson, students used place value understanding to subtract 3-digit numbers. Students also connected place value understandings to a standard algorithm for subtraction. **Developing the Big Idea:** Students further *develop* the understanding of a standard algorithm as a shortcut for the expanded algorithm. This is achieved by connecting place value understanding to subtracting multi-digit numbers where they have to regroup across zero.	**Instructional Note:** The majority of problems in this lesson are compare problem types which research indicates tend to be more difficult for students. Watch for students that seem to be struggling due to the comparative terminology and provide vocabulary support for the terms. Focus on making meaning of the problems and determining appropriate plans. **Solve & Share:** Consider giving students opportunity to solve the problem any way they choose. Have tools readily available for students to use if needed. Therefore, it is acceptable for students to be using expanded or partial differences algorithms to solve multi-digit subtraction problems. After introducing the *Solve & Share*, consider asking students if there is any information we do not need to solve the problems (e.g. Rick is allowed to receive 1,000 texts per month). If students do not offer a solution method that is the same as "Sam's Work", then consider discussing "Sam's Work" as a class (TE p. 511). Notice that Sam offers a visual representation and a written explanation of the regrouping. **Look Back:** Consider assigning the *Look Back!* to support students' understanding of the inverse relationship between addition and subtraction. **Visual Learning:** The *Visual Learning Animation* only shows the U.S. Traditional Algorithm as a solution method. Consider pausing frequently to connect the computation with the standard algorithm to partial sums, including using base-10 blocks if needed. You might also consider not showing the *Visual Learning Animation*, but still have students solve and share their invented algorithms, or partial differences, with place value understanding. **Convince Me:** If you do show the *Visual Learning Animation*, consider assigning and discussing the *Convince Me!* to help students see the place value connections to the U.S. Traditional Algorithm. **Independent Practice/Math Practices and Problem Solving:** *Quick Check* item 19 requires students to reason with a multi-step compare problem type. Consider asking students what the hidden questions are in order to solve this problem. ***-continues on next page-***

		Assess and Differentiate: If time permits, teach students how to play "Display the Digits" (TE, p. 515A). All students should have the opportunity to play this game. Child watch to identify students who need additional support and consider the included *Intervention Activity* (TE, p. 515A). Consider utilizing the following question format during practice: **Example Stem 1:** What unknown number makes this equation true? 763 – 97 = 763 – 100 + □ Enter your answer in the response box. **Rubric:** The student enters the correct number to make the equation true (e.g., 3). **Example Stem 2:** What unknown number makes this equation true? 760 – 70 = 760 – 60 – □ Enter your answer in the response box. **Rubric:** The student enters the correct number to make the equation true (e.g., 10).
Lesson 9-8: Math Practices and Problem Solving- Construct Arguments		
3.NBT.A.2 **MP.3** MP.1 MP.2 MP.4 MP.7	**Access Prior Learning:** In Grade 2, students learned how to explain their thinking when proving answers to addition problems. **Developing the Big Idea** In this lesson, students further *develop* their understanding of MP. 3 *"Construct viable arguments and critique the reasoning of others"* using addition and subtraction to justify conjectures.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 3, *"Construct viable arguments and critique the reasoning of others*." Refer to the *Math Practices and Problem Solving Handbook* for suggestions on how to develop, connect and assess this Math Practice (TE, pp. F23-F23A, F29). Also, reference the handbook in the Student Edition (SE, p. F23). **Solve & Share:** Consider reintroducing MP. 3 Thinking Habits (SE, p. F23) before introducing the *Solve & Share*. Watch for students that are able to complete the task, but are unable to explain how they know they have the largest sum. Consider supporting these students with questioning about how they can see and used place value in their strategies? **Look Back:** Consider discussing as a class the *Look Back!* question to support students' mathematical reasoning skills and place value understandings. **Convince Me:** Consider assigning the *Convince Me!* as it provides an opportunity for students to reason more with MP.3 by supporting a conjecture with a representation. **Assess and Differentiate:** If time permits, you may consider replacing the *Problem Solving Reading Mat* with the game "Clip and Cover" (TE, p. 485A), "Teamwork" (TE, p. 491A), "Display the Digits" (TE, p. 497A), "Display the Digits" (TE, p. 515A), or the Fluency Practice Activity (TE, p. 523). Child watch to identify students who need additional support and consider the included *Intervention Activity* (TE, p. 521A).

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Rolling for 500

Mathematical Understanding:
Students strengthen numerical fluency through practice with strategies used for addition and subtraction.

Grade Level: 3-5

Number of Players: 2-4

Materials Needed:
- a die
- a game piece for each player
- game board

Object of the Game:
The first player to reach or cross the **Finish** wins the game.

Directions:
Each player places their marker on the **Start** square of the shared game board.

Player 1 rolls the die. Match the number rolled to the table on the game board to determine how many spaces to move forward or backward. Player 1 moves their marker.

Players take turns rolling the die and using the table to determine spaces moved.

The first player to reach or cross the **Finish** line wins the game.

Players cannot move below zero and wait at the start space for a positive roll.
Two players can be on the same space on the game board at the same time.

Optional:
When playing the estimation version, players can state aloud what their exact space would be and how close they are to the space they move onto to. Which space is the closest and why

Guiding Questions:
What do you know?
Where do you think you will begin?
Where are you stuck? What is confusing? What are you wondering about?
What are you going to try?
What did you think about to come to your answer?

Differentiation:

Two versions of the game can be used for grades 3-5. **Rolling for 500** gives practice with place value strategies to add and subtract numbers up to 500. **Rolling for 500 estimation** gives practice with place value strategies for addition and subtraction and also requires comparative reasoning in order to properly place the game board marker.

Game Trajectory:
Pre K-K: Counting along a number line to 20

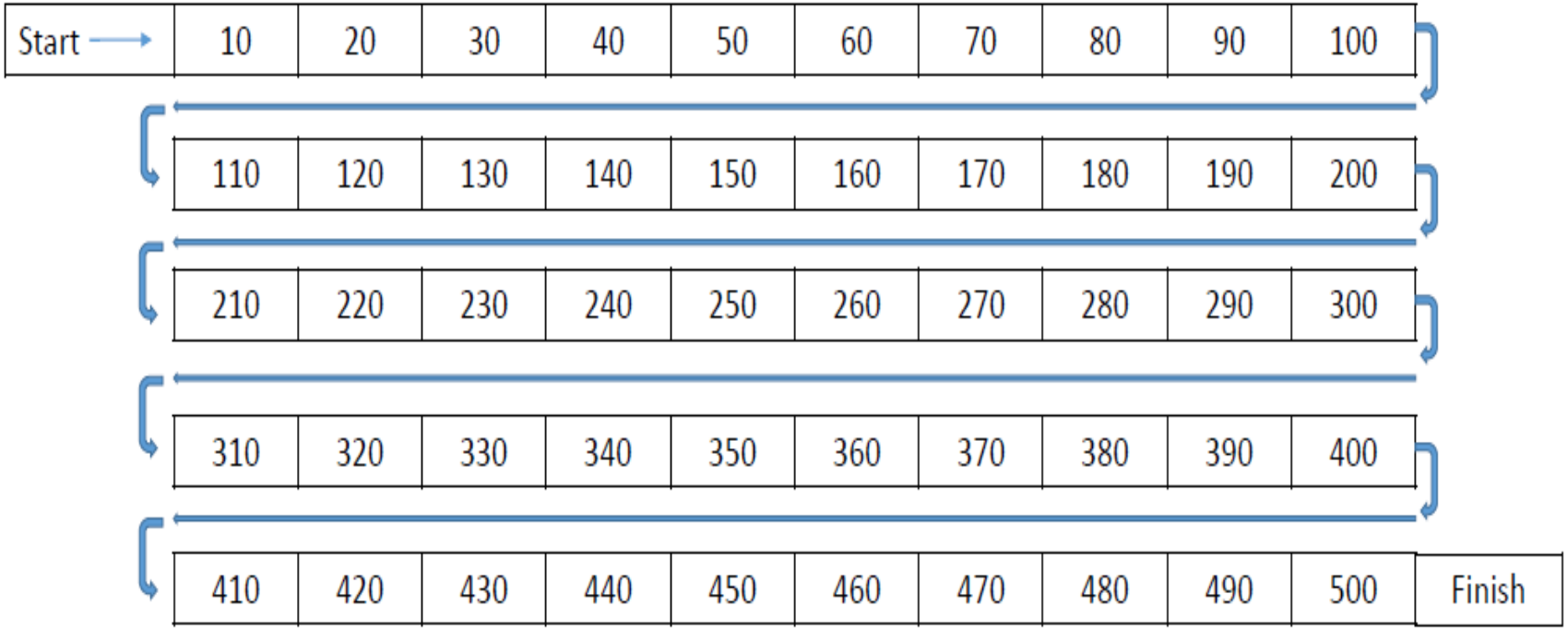
K-2: Addition and subtraction to get to 50

3-5: Rolling for 500 or Rolling for 500 estimation version

Clean up Checklist for Game Bag:
Die
Game piece markers
Game boards

Rolling for 500

Roll	Spaces
1	add 30
2	subtract 20
3	add 50
4	subtract 60
5	add 80
6	add 10



Grade 3 Topic 15: Attributes of Two-Dimensional Shapes

Topic 15
Attributes of Two-Dimensional Shapes
Number of lessons: **4**
F/D/E: **7** days
NVACS Focus: G.A
Total Days: ~11

[3rd Grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: K-6, Geometry (pp. 13-14)

Prior to instruction, view the Topic 15 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 805A-805F), the Topic Planner (pp.805I-805J), all 4 lessons, and the Topic Assessments (pp. 841-842A).

Mathematical Background: Read Topic 15 Cluster Overview/Math Background (TE, pp. 805A-805F)	**Topic Essential Question:** How can two-dimensional shapes be described, analyzed, and classified? *Reference Answering the Topic Essential Question (TE, pp. 839-840) for key elements of answers to the Essential Question.*

The lesson map for this Topic is as follows:

15-1	15-2	15-3	15-4	Assessment

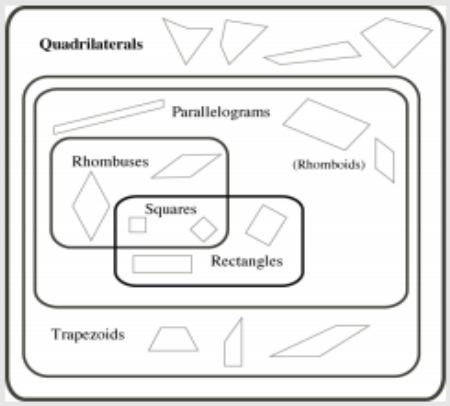
7 F/D/E days used strategically throughout the topic includes the "enrichment" lessons.

Instructional note:

Topic 15's big idea is that two-dimensional shapes can be described, analyzed, and classified based on their attributes. Students learn to analyze a variety of geometric shapes and begin to explore relationships between the shapes based on their attributes. The Geometry progression document (2014) describes attributes as, "any characteristic of a shape, including properties, and other defining features (e.g., straight sides) and non-defining features (e.g., "right-side up")" (p. 3).
Much of what makes this standard a struggle for students is the vocabulary and hierarchical inclusion of geometric shapes (any shape in a sub category is also a member of the larger category; yet a shape of a larger category may or may not be part of particular sub categories).

Geometric hierarchy is the idea that, "Shapes have many attributes that make them similar to and different from one another. You can describe and classify different groups of shapes by their attributes" (TE, p. 818). This means that a shape can fall into many different groups when being classified. For example, when classifying a square, a square meets the requirements of being a polygon, quadrilateral, rectangle, parallelogram, and a rhombus. However, a rectangle and a rhombus are not necessarily squares.

Venn diagram showing classification of quadrilaterals



Note that rhomboids are parallelograms that are not rhombuses or rectangles. This example uses the inclusive definition of trapezoid (see p. [pageref "T(E)"]).

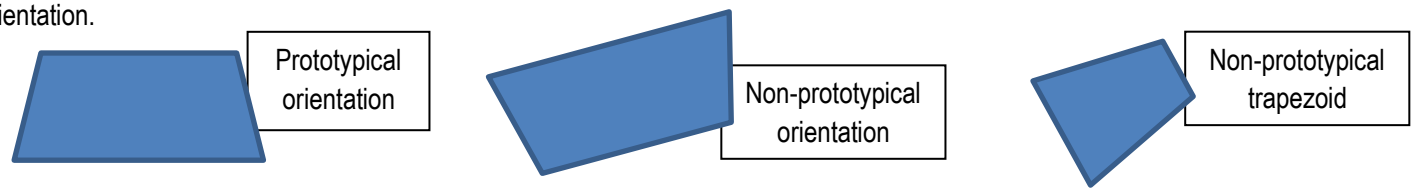
To illustrate this idea, please see the image to the right from page 18 of the K-6, Geometry progression document (2014). Additional A/D/E days allow instructional time for tasks that build understanding of the idea that shapes can be classified into a group that is part of another group. Include non-examples to help students establish limitations to categories.

Vocabulary becomes an important component of recognizing and classifying shapes by their attributes. This can be illustrated by the example of classifying a square which moves through several subcategories and also retains the attributes and names from those subcategories. For example, to be labeled a square, students have to recall that the shape must also meet all of the following criteria:

- be a polygon (and the all the attributes for a polygon)
- have four sides
- have 2 pairs of parallel sides
- all sides are the same length
- all angles are the same size

Students who have had limited experiences with working with geometric ideas in previous grades may need additional language support. Consider using the graphic organizers from Teaching Tools 24 through 28 found in the *Teacher's Resource Masters Volume 2* to support language acquisition and use. Anchor charts, cognitive content dictionaries or personal word walls in addition to writing and reasoning tasks will support students in use of the academic language needed to explore these ideas.

A common misconception that students will form is that a shape is only that shape when presented in its prototypical orientation. For example, the first trapezoid below is shown in its prototypical orientation, while the second trapezoid is shown a non-prototypical orientation.



To avoid this misconception, provide many opportunities for students to reason with polygons in multiple orientations and name polygons based on evidence of the attributes stated in the shape's definition.

There are two different definitions for a trapezoid, an inclusive and **exclusive** definition. Per the Geometry progression document (2014) the inclusive definition of a trapezoid states that, "a trapezoid is a quadrilateral with **at least one pair** of parallel sides" (p.3). Therefore, in the inclusive definition, a trapezoid would fit into the sub-category of parallelograms. In the Geometry progression document (2014) the exclusive definition of a trapezoid states that, "a trapezoid is a quadrilateral with **exactly one pair** of parallel sides" (p.3). Therefore, in the **exclusive definition,** a trapezoid would not fit into the sub-category of parallelograms. There is no harm in students being made aware of the two different definitions. However, **enVision**math**2.0** and WCSD use the exclusive definition (K-Algebra 1).

The WCSD 3rd Grade Pacing Framework allows an additional 7 days of lessons to explore geometric ideas and build a solid understanding that will be necessary for students as they progress through the grade levels. To assist in building strong conceptual development and support learner responsive instruction, additional "Enrichment" lessons are included throughout the topic. More ideas are found at the conclusion of this document.

Focus Math Practice 6: **Attend to precision**
Focus on opportunities for students to develop Mathematical Practice 6 behaviors throughout the entire topic, as this is the focus of the Math Practices and Problem Solving lesson 15-4. Resources to support students' development of MP. 6 include the Teacher's Edition (pp. F26 - F26A) and the Nevada Academic Content Standards for Mathematical Practice.

Looking ahead to the Topic Performance Assessment, students need to be familiar with the attributes of two-dimensional figures and be able to identify shared attributes among grouped two-dimensional figures. Develop thinking habits that allow students to engage in these types of problems through encouraging discussions, deliberations and debates while having students work with these ideas in triads (groups of 3).

Essential Academic Vocabulary Use these words consistently during instruction.		
New Academic Vocabulary: (First time explicitly taught)		**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
polygon	parallelogram	*circle*
side	rectangle	*hexagon*
quadrilateral	right angle	*pentagon*
angle	rhombus	*triangle*
vertex	square	*rhombus*
trapezoid	convex	*rectangle*
parallel sides	concave	

Additional terminology that students may need support with: attributes, alike, different, square angle

***Collaborative Team Conversations (CTC)**
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

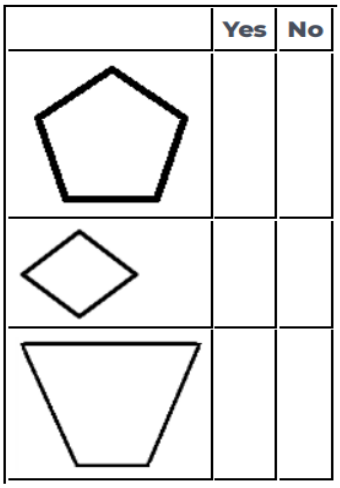
Guiding questions: "Are students able to classify and analyze quadrilaterals based on their attributes?"

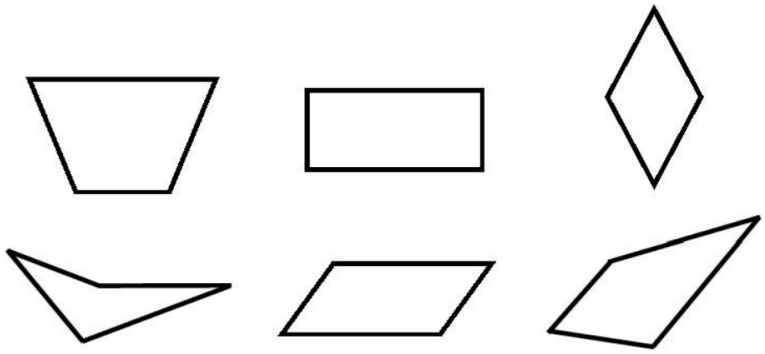
Lesson	Evidence	Look for
15-3	***Quick Check*** (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources". • students are able to analyze and compare quadrilaterals based on their attributes.
15-4	***Convince me!*** (student work samples)	Focus CTC around the big idea: • construct shapes within given parameters.

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 839-842	Use *Scoring Guide* TE pp. 839-842A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Optional Enrichment		
3.G.A.1 MP.1 MP.3 MP.6 MP.7	**Access Prior Learning**: In Topic 15, Grade 2, students learned about polygons and angles. Students have been working with prototypical and non-prototypical shapes and defining attributes since pre-K or Kindergarten. **Developing the Big Idea:** Students further *develop* geometric concepts by identifying common attributes amongst shapes (both defining and non-defining).	**Materials:** Attribute Blocks (these may be checked out from the UNR LRC) Shape clues (attributes of various shapes that may fall in various subcategories). **Opener:** Students work in triads to create an attribute train. One player starts by picking a block. The next player must pick a block that is different in only one way. The difference can be shape, size, thickness or color. The triad keeps taking turns building their train and checking that each car is different in only one way. Once all blocks have been placed rotate the triads so that one triad checks another triad's train for accuracy. Play again, this time choose blocks that change ***in all but one*** attribute. That means that cars that touch must have only one attribute in common. **Whole Group Discussion:** After the second version of the game has been played, choose one group's work to discuss whole group. Consider using a fishbowl strategy to explore the train. Highlight attributes of shapes. Extend attribute thinking by giving students clues (attributes of a shape) and having them draw the shape (2nd grade standard). Example: I have four equal sides and four equal corners (this could be a square or a rhombus). Purposely choose attributes that could be used to make shapes that may fit more than one category. See "Polygons on the Geoboard" at the end of this guide for clue ideas. Encourage children to visualize or mentally construct the shape as they are drawing the shape. Create an anchor chart that lists the attributes of triangles, quadrilaterals, pentagons and hexagons (2nd grade standard). Add on to this chart throughout the topic starting with 15-1. Note: "Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes" (1st grade NVAC, 1.G.A.1).
Lesson 15-1: Describe Quadrilaterals		
3.G.A.1 MP.1 MP.3 MP.4 **MP.6** MP.7 MP.8	**Access Prior Learning**: In Topic 15, Grade 2, students learned about polygons and angles. Students have been working with prototypical and non-prototypical shapes and defining attributes since pre-K or Kindergarten.	**Topic Opener:** Introduce the *Topic Essential Question*, "How can two-dimensional shapes be described, analyzed, and classified?" (TE p. 805). Consider making an anchor chart highlighting key ideas so that students can see the conceptual development and connections throughout the topic. Have students complete the *Review What You Know* prior to beginning instruction on Topic 15 so that you can respond to students' instructional needs. Some students may benefit from additional support using the *Item Analysis for Diagnosis and Intervention* prior to beginning the topic (TE, p. 806). Consider introducing vocabulary as students encounter terminology in the lessons rather than introducing all terms at the beginning of the lesson (avoid front loading). ***-continues on next page-***

	Developing the Big Idea: Students further *develop* geometric concepts by describing and classify different quadrilaterals by their sides and angles.	**Solve & Share:** Consider providing copies of various quadrilaterals (Teach Tool 21). After introducing the *Solve & Share*, discuss the questions provided in the section *Build Understanding* to help students develop a possible strategy for solving (TE, p. 811). Consider adding non-prototypical shapes. For more details, see the *Instructional note* at the beginning of this topic. **Look Back!**: After discussing students' solution methods and reasoning, discuss the *Look Back!* prompt if those ideas do not already come out during the whole class discussion. **Visual Learning:** Consider pausing and discussing after the *Visual Learning Animation* poses the question, "Why are all of these shapes quadrilaterals?" **Independent Practice/Math Practices and Problem Solving**: Consider assigning items 11 and 12 "Vocabulary" to provide students with additional opportunities to develop the mathematical language in this topic. Consider utilizing the following question format during practice: Example 1 Full Statement **Example Stem:** Decide if each shape is a quadrilateral. Select Yes or No for each shape.  **Rubric:** (1 point) The student correctly identifies each shape as Yes or No (e.g., N, Y, Y).
Enrichment		
3.G.A.1 MP.1 MP.3 MP.6	**Access Prior Learning**: In Topic 15, Grade 2, students learned about polygons and angles. Students have been working with prototypical and non-prototypical shapes and defining attributes since pre-K or Kindergarten. **Developing the Big Idea:** Students further *develop* geometric concepts by describing and classify different quadrilaterals using attributes.	**Materials:** Assorted Shape BLM 44,45 and 46 from: https://tinyurl.com/Topic-15-Shapes **Opener:** Students work in triads to cut out the shapes from the above masters. Students work as a group to sort the shapes into several categories (open sort of at least four categories). Once all the shapes have been placed into groups, have students rotate to another triad's sort. Have this triad analyze the classification by labeling each group of shapes. Have students create a list of all the attributes the shapes in the group have in common. Ask triads to write a statement on if they agree or disagree with the sort and why. **Whole Group Discussion:** Choose one triad's work to discuss whole group. Highlight attributes of shapes. When analyzing parallel lines, consider using two straight edges (meter sticks or rulers) to test lines that students think are parallel, yet end up intersecting (when the straight edges are placed on either supposedly parallel line to show the lines will eventually intersect even if they are not intersecting now).
Lesson 15-2: Classify Shapes		
3.G.A.1 MP.3 MP.5 **MP.6** MP.7 MP.8	**Access Prior Learning**: In the previous lesson, students explored classifying shapes using their attributes. This builds from 2nd grade when students recognized and drew space when given specified attributes.	**Instructional note:** In third grade, students do not formally measure angles. Students classify based on size of angles from "eye balling" right/square angles. **Solve & Share:** Consider assigning the *Look Back!* to extend students' reasoning of how they sorted the triangles. ***-continues on next page-***

	Developing the Big Idea: Students further *develop* geometric concepts by classifying shapes by the number of sides, equal sides, parallel sides, size of angles, and concavity versus convexity.	**Visual Learning:** During the *Visual Learning Animation*, consider pausing and discussing the questions: • "How are the groups different?" • "How are the groups alike?" After viewing the *Visual Learning Animation* consider discussing the prompt provided in *Prevent Misconceptions* (TE, p. 818). Assign the *Convince Me!* to check for understanding of the *Visual Learning* Animation and extend the ideas presented. **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 16 "Algebra" to provide distributed practice of "Put Together/Addend Unknown" (see page 88 of NVACS, 2010 for more information of problem types) problem type and algebraic reasoning. **Assess & Differentiate:** Consider providing an opportunity for all students to interact with the *Math and Science Activity* (TE, p. 821A). This activity provides a meaningful experience with identifying and naming polygons based on their attributes. Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 821A). Use triangles from master cutouts (see above lesson). Consider utilizing the following question format during practice: Example 1 Full Statement **Example Stem:** Select **all** of the shapes that appear to be parallelograms.  **Rubric:** (1 point) The student correctly selects all of the parallelograms (e.g., see image below).
Enrichment		
3.G.A.1 MP.1 MP.3 MP.6	**Access Prior Learning**: In the previous lesson, students explored classifying quadrilaterals by using their attributes. **Developing the Big Idea:** Students further *develop* geometric concepts by classifying shapes into formalized categories in this case parallelograms and quadrilaterals.	**Materials:** Assorted Shape BLM 44,45 and 46 from: https://tinyurl.com/Topic-15-Shapes (see enrichment after 15-1). Parallelogram vs. Quadrilateral sorting mat: https://tinyurl.com/Topic-15-sort-mat **Opener:** Students work in partners or triads to sort the shapes using the Parallelogram vs. Quadrilateral sorting mat. After about 6 minutes of exploration, pull the group together and have students share the attributes of a parallelogram. Post or list these on an anchor chart for students to reference. Post the word quadrilateral and have students discuss what a quadrilateral is in their triads (yet don't post attributes yet).Have students analyze their work so far and then continue working. **Whole Group Discussion:** Have students do a gallery walk to look at the other triads work. Bring students together to discuss their findings. What did students find? What is interesting about this sort? Can they think of a shape that could go into just the parallelogram spot (that would not fit both or either too)? Highlight attributes of shapes. Use two straight edges (meter sticks or rulers) to test lines that students think are parallel. Full lesson available at: https://tinyurl.com/Topic-15-enrichment-lesson)

Lesson 15-3: Analyze and Compare Quadrilaterals

3.G.A.1
3.MD.C.5b

MP.2
MP.3
MP.4
MP.7

Access Prior Learning:
In previous lessons, students described and classified quadrilaterals.

Developing the Big Idea:
Students further *develop* geometric concepts by analyzing and comparing quadrilaterals and group them by their attributes.

Instructional note:
A common struggle for students is recognizing that while all squares are rhombuses not all rhombuses are squares. Consider using an ADE day to develop a class anchor chart that represents the hierarchical inclusion of these shapes. This idea refers to the hierarchical inclusion of geometric shapes, for more details on this read the *Instructional Note* at the beginning of this topic.

Solve & Share:
After introducing the *Solve & Share* consider asking the questions provided in *Build Understanding* activate prior learning.

Consider assigning the *Look Back!* prompt to extend student reasoning from the *Solve & Share*.

Visual Learning:
Consider pausing and discussing after the *Visual Learning Animation* poses the question, "How do you know all the shapes are quadrilaterals?" Consider assigning the *Convince Me!* to check for understanding of the *Visual Learning Animation* and extend the ideas presented.

Independent Practice/Math Practices and Problem Solving:
Consider having students work in partners and triads and to reason about and discuss the problems. Encourage students to discuss, debate, define possible solutions using appropriate and precise mathematical terminology.

Assess & Differentiate:
Child watch to identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p. 827A).

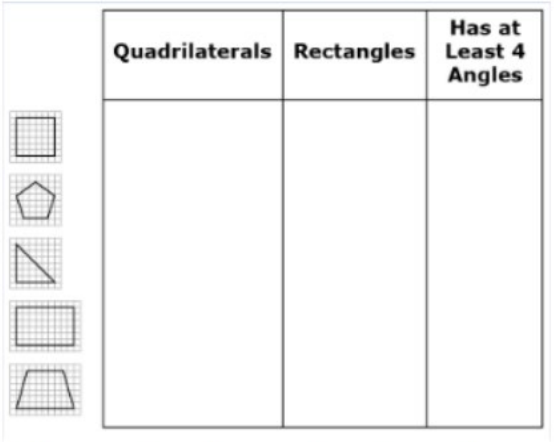
CTC: *Quick Check (digital platform)

Consider utilizing the following question format during practice:

Example 1
Full Statement

Example Stem: Drag the figures to each box or boxes where they belong.

A figure may belong to more than one category or to none of these categories.

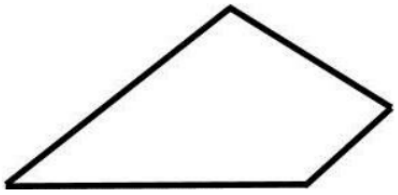


Rubric: (1 point) The student correctly classifies each shape (e.g., see chart below).

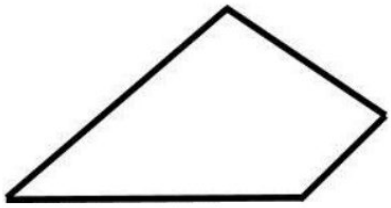
Example 2
Full Statement

Example Stem 2: Use the Connect Line tool to draw a quadrilateral where every side is a different length.

Rubric: (1 point) The student correctly draws a quadrilateral that meets the given attributes (e.g., see quadrilaterals below)



-continues on next page-

		Example 3 Full Statement **Example Stem 3:** Use the Connect Line tool to draw a quadrilateral that is **not** a rhombus or a rectangle. **Rubric:** (1 point) The student correctly draws a quadrilateral that meets the given attributes (e.g., see quadrilaterals below) 
Lesson 15-4: Math Practices and Problem Solving- Precision		
MP.6 MP.1 MP.3 MP.5 MP.7 **3.G.A.1**	**Access Prior Learning**: In previous lessons, students described, classified, analyzed, and compared quadrilaterals. **Securing the Big Idea:** Students *secure* the geometric concepts of describing, classifying, analyzing, and comparing quadrilaterals by using accurate language and using appropriate tools.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 6. Refer to the *Math Practices and Problem Solving Handbook* for suggestions on how to develop, connect and assess this Math Practice (TE, pp. F26-F26A, F29). Also reference the handbook in the Student Edition (SE, p. F26). **Solve & Share:** Consider reintroducing MP. 6 Thinking Habits (SE p. F26) before introducing the *Solve & Share*. Also consider using time students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.6, that are listed in the *Math Practices and Problem Solving Handbook* (TE, p. F26A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice. Have grid paper, rulers, and index cards (or other tools for making right angles) available. After introducing the *Solve & Share*, consider discussing the questions provided in the section *Build Understanding* to help students develop a possible strategy for solving (TE, p. 829). After discussing students' solution methods and reasoning, consider discussing the *Look Back!* prompt to support students' development of MP.6. **Assess & Differentiate:** Consider providing an opportunity for all students to interact with the Math and Science Activity (TE, p. 833A). This activity provides a meaningful experience with identifying and naming polygons based on their attributes. Child watch to identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p. 833A). ***CTC: *Convince Me!*** (student work samples)

References

Common Core Standards Writing Team. (2013, September 19). *Progressions for the Common Core State Standards in Mathematics (draft). K-6, Geometry.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Wiest, L. (2015). *Lecture 3: Polygons.* Reno, NV: University of Nevada, Reno

Van De Walle, J. A., Bay-Williams, J. M., Lovin, L. H., & Karp, K. S. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 3-5* (2nd ed.). New York, NY: Pearson.

Additional enrichment activities:

- Graham Fletcher's "Geo-Dotting" (DOK 2 potential when incorporated into whole class discussion)
- Shape-Match (DOK 1, limited to prototypical standard orientations)
- Quadrilateral Riddle Creator (DOK 1)
- Quadrilateral & Polygon Sort (DOK 2)

Polygons On The Geoboard

1) Make a 3-sided polygon with 1 square corner and no 2 sides the same length.

2) Make a 4-sided polygon with no parallel sides.

3) Make a 4-sided polygon with all sides different lengths.

4) Make a 4-sided polygon with no square corners but with two pairs of sides parallel.

5) Make a 5-sided polygon that has exactly one pair of parallel sides.

6) Make a 6-sided polygon with three pairs of parallel sides.

7) Make a 6-sided polygon with one pair of sides perpendicular.

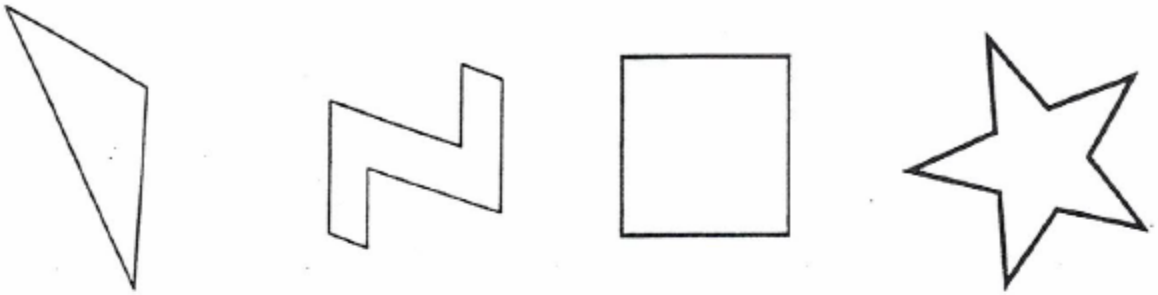
8) Make a polygon that is not a square and looks the same no matter on which side you rest the geoboard.

9) Make a polygon with as many sides as is possible on the geoboard.

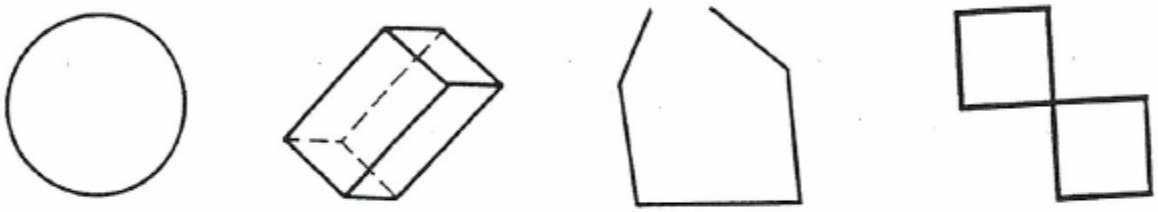
25

* To incorporate technology and encourage discussion on 2-dimensional shapes consider using, https://apps.mathlearningcenter.org/geoboard/.

These are polygons.

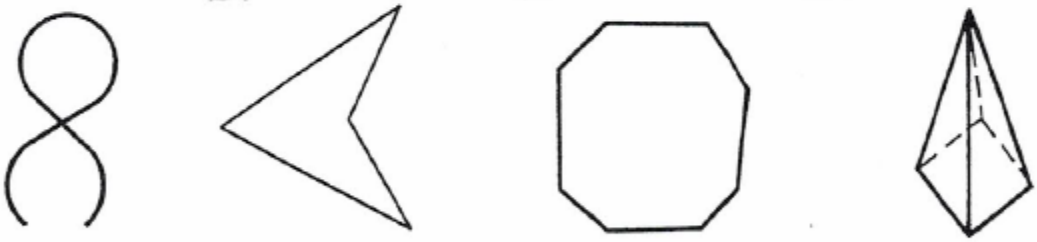


These are not polygons.



Which of these are polygons?

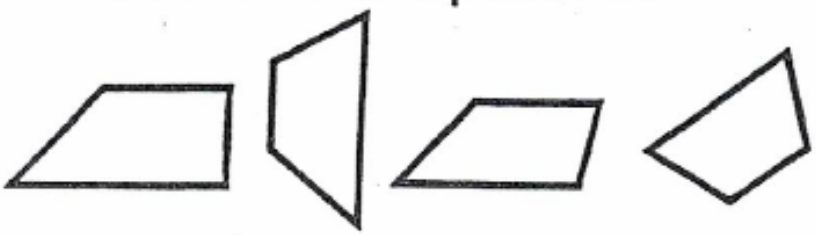
A. B. C. D.



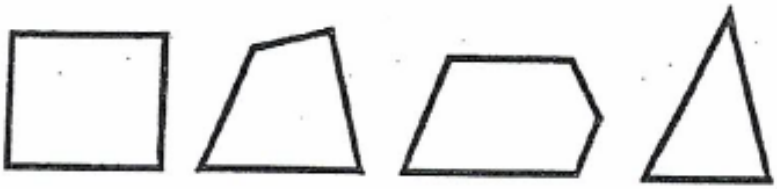
Draw some polygons.

Define "polygon."

These are trapezoids:



These are not trapezoids:



Which of these are trapezoids?

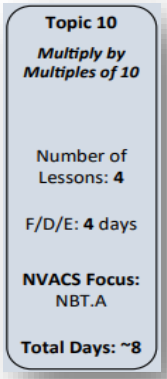
a.  b.  c.  d. 

Draw something that is a trapezoid.

Draw something that is not a trapezoid.

What is a trapezoid?

Grade 3 Topic 10: Multiply by Multiples of 10



3rd Grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Numbers and Operations in Base Ten, K-5 (p. 12)

Prior to instruction, view the Topic 10 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 571A-571F), the Topic Planner (pp.535A-535B), all 4 lessons, and the Topic Assessments (pp. 569-570A).

Mathematical Background: Read Topic 10 Cluster Overview/Math Background (TE, pp. 571A-571F)	**Topic Essential Question:** What are ways to multiply by multiples of 10? *Reference Answering the Topic Essential Question (TE, pp. 567-568) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

10-1	10-2	10-3	10-4	Assessment

4 F/D/E days used strategically throughout the topic

Instructional note:

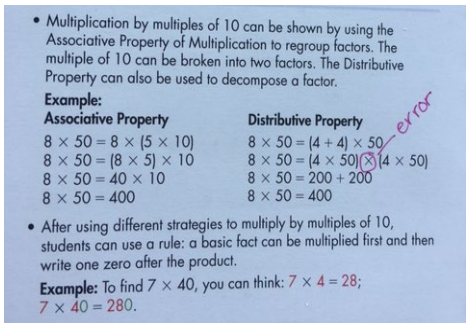
In Topics 1 through 5 students developed conceptual understanding of multiplication and division. These critical mathematical understandings come together in Topic 10 to develop understanding of multiplying by a multiple of 10. Topic 10 is part of a topic cluster with Topics 8 and 9 that share the big idea of using place-value understanding and properties of operations to perform multi-digit arithmetic. A big idea of Topic 10 is exploring the place value patterns seen with 10s to build a deeper understanding of place value and build number sense. This understanding will be critical in establishing the "write a zero" rule that is the focus of lesson 10-3.

It is important that the work in this topic not be minimized to having students memorize that they can write a zero and multiply the remaining digits. Teaching the "zero trick" without developing the mathematical understanding behind the rule creates misconceptions when students need to generalize this understanding to working with larger multiples of 10, when confronted with a zero in the middle of a number (ex.6,402) and when students begin working with decimals. Students should understand that a place value is being added to a number when it becomes 10 times greater because of the base 10 place value system. The added zero is holding the added place value.

To help develop this understanding, many learners need to model groups of 10 with the base-ten blocks. For example, in lesson 10-2 the Associative and Distributive Property of Multiplication are used to decompose the multiple of 10. In the *Visual Learning Animation,* the 20 is decomposed into 2 x10 so that the expression 4 x 20 becomes 4 x 2 x 10. Allowing students to build the two different but equivalent expressions with base-ten blocks makes these ideas more accessible and concrete for all students.

Focus Math Practice 7: Look for and make use of structure

The standard states, "Mathematically proficient students look closely to discern a pattern or structure" (NVACS, 2010, p. 8). To help students work towards security, consider connecting ideas for the "write a zero" rule to our base-10 place value system. Behaviors associated with MP.7 are described in the Teacher's Edition (TE, pp. F27 - F27A) and the Nevada Academic Content Standards for Mathematical Practice.



- Multiplication by multiples of 10 can be shown by using the Associative Property of Multiplication to regroup factors. The multiple of 10 can be broken into two factors. The Distributive Property can also be used to decompose a factor.

Example:

Associative Property	Distributive Property
$8 \times 50 = 8 \times (5 \times 10)$	$8 \times 50 = (4 + 4) \times 50$
$8 \times 50 = (8 \times 5) \times 10$	$8 \times 50 = (4 \times 50) \times (4 \times 50)$ error
$8 \times 50 = 40 \times 10$	$8 \times 50 = 200 + 200$
$8 \times 50 = 400$	$8 \times 50 = 400$

- After using different strategies to multiply by multiples of 10, students can use a rule: a basic fact can be multiplied first and then write one zero after the product.

Example: To find 7×40, you can think: $7 \times 4 = 28$; $7 \times 40 = 280$.

Looking ahead to the Topic Performance Assessment, students will have to apply strategies for division to answer item 6. Topic 4 developed students' ability to reason and solve for division situations without having to formally divide.

Finally, please note there is an error in the Teacher's Edition on page 567. The error is indicated in the image; the multiplication symbol should be an addition symbol.

Meaningful Fluency Practice & Assessment:

For students to attain security with NVACS 3.OA.C.7 and 3.NBT.A.2 it is critical that the established meaningful fluency practice and assessment practices continue. Refer to Topic 1 for details about meaningful fluency practice and assessment practices. Topics 1-5 include games for meaningful fluency practice for multiplication and division (NVACS 3.OA.C.7). Topics 8 and 9 include games for meaningful fluency practice for multi-digit addition and subtraction within 1000 (NVACS 3.NBT.A2). The following game will support students developing understanding of multiplying with 10s and provides sentence frames to support language development necessary for explanations.

Phase 3: Multiply by Multiples of 10

Materials: set of cards (0-9)

Gameboard (one for each player)
Sentence frames (one for each player)
Counters and/or Base-10 blocks to support student understanding

Directions: Shuffle the cards and place them face down in a stack. Each player flips over two cards from the top of each stack and places the cards on the empty boxes on the gameboard (at the end of this document) to make the multiplication equation. Each player solves their own equation and explains their thinking, using the sentence frames (at the end of this document), if needed. The player with the largest product earns 1 point. Play continues until a player earns 10 points.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
open number line (10-1)	*equation* *product* *multiple* *Associative Property of Multiplication* *Distributive Property of Multiplication*

Additional terminology that students may need support with: *pattern, relationship, basic fact (see 10-3* ***Visual Learning Animation*** *for details)*

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "How are students applying place value understanding to add and subtract whole numbers?"

Lesson	Evidence	Look for
10-2	***Solve and Share!*** (student work samples)	Focus CTC around the big idea: • look for students who explain the properties • differences and similarities between student examples • applying multiplication to multiples of 10 and use of basic facts
10-4	***Quick Check*** (digital platform) Items 1 and 3	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources". • understanding patterns based on multiples of 10

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 567-570	Use *Scoring Guide* TE pp. 567-570A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	**Mathematical Development of the Big Idea**	**Instructional Clarifications & Considerations**
Lesson 10-1: Use an Open Number Line to Multiply		
3.NBT.A.3 MP.2 MP.4 **MP.7** MP.8	**Access Prior Learning:** In Topic 1, students learned how to use number lines to show multiplication and that multiplication is the joining of equal groups. **Developing the Big Idea:** Students *begin* to understand strategies for multiplying by a multiple of 10 by showing the multiplication on an open number line.	**Topic Opener:** Introduce the *Topic Essential Question*, "What are ways to multiply by multiples of 10?" (TE p. 535). Consider using this question to make an anchor chart with your student. As new ideas are added during the topic, students will see the development of ideas and make connections. You might also consider having students complete the *Review What You Know* prior to beginning instruction on Topic 10 so that you can respond to students' instructional needs using the *Item Analysis for Diagnosis and Intervention* (TE, p. 536-537). Consider introducing vocabulary as terms are encountered in the lessons rather than introducing all terms at the beginning of the lesson. **Solve & Share:** The questions provided in the *Build Understanding* (TE, p. 539) help students access prior learning in the conventions of using an open number line to show multiplication. Watch for students struggling to find an appropriate strategy and consider asking these questions to scaffold as students are working. ***-continues on next page-***

		If students do not offer a solution method similar to "Alex's Work", then consider discussing "Alex's Work" as a class (TE, p. 539). Alex's work shows an example of using repeated addition to solve for multiplication which helps to make the reasoning accessible to all students. To support students' development of MP. 7, consider discussing the *Look Back!* prompt and, if necessary, providing students with a multiplication table to help them develop a conjectures for patterns with the 2's facts, 10's facts, and the facts they solved for in today's *Solve & Share*. **Independent Practice/Math Practices and Problem Solving:** Consider assigning and discussing item 9 to help students develop schema for patterns that can be used when multiplying with 10s. This problem also offers language that can support students' ability to connect multiplying facts they know to multiplying with 10s. **Assess and Differentiate:** If time permits, you may consider replacing the Math and Science Activity with games from previous topics or the *Fluency Practice Activity* (TE, p. 563). Child-watch to identify students who need additional support and consider the *Intervention Activity* provided (TE, p. 543A). Consider utilizing the following format during practice: **Example Stem 1:** What unknown number makes the equation true? 5 x 80 = □ Enter your answer in the response box. **Rubric:** (1 point) The student enters the correct product (e.g., 400).
Lesson 10-2: Use Properties to Multiply		
3.NBT.A.3 MP.1 MP.3 MP.6 **MP.7**	**Access Prior Learning:** In Topic 3, students learned the Associative and Distributive Properties of Multiplication. **Developing the Big Idea:** Students are further *developing* their understanding of strategies for multiplying by a multiple of 10 by using their understanding of place value and the properties of multiplication.	**Solve & Share:** Watch for students that say that Earl's response is incorrect because he starts his argument with an equation that shows the product on the wrong side (e.g., 30 = 3 x 10). These students are misinterpreting the equal sign as a symbol for "the answer goes here" and need support on understanding the equal sign as a symbol that communicates equivalence. Consider discussing the *Look Back!* prompt as a key idea to understanding the use of the Associative Property of Multiplication to get a basic fact. **Visual Learning:** Consider pausing and discussing strategies to answer, "How can you find the product 4 x 20?". Consider assigning and discussing the *Convince Me!* to give students the opportunity to reason with the Associative Property of Multiplication after it's been applied. **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 16 *Number Sense* to support students' development of number sense and the application of the Associative Property of Multiplication to reason with numbers. **Assess and Differentiate:** If time permits, you may consider replacing the *Problem Solving Reading Mat* with games from previous topics or the *Fluency Practice Activity* (TE, p. 563). Child-watch to identify students who need additional support and consider the *Intervention Activity* provided (TE, p. 549A). ***CTC: *Solve and Share!*** (student work samples) Consider utilizing the following format during practice: **Example Stem:** What unknown number makes the equation true? (6 x 5) x □ = 240 Enter your answer in the response box. **Rubric:** (1 point) The student enters the correct value for the unknown (e.g., 8). ***-continues on next page-***

Lesson 10-3: Multiply by Multiples of 10		
3.NBT.A.3 MP.1 MP.3 MP.4 MP.5 **MP.7** MP.8	**Access Prior Learning** In the previous lesson, students used the properties of multiplication to multiply by a multiple of 10. **Securing the Big Idea** Students are *securing* their understanding of strategies for multiplying by a multiple of 10 by using their understanding of place value and the properties of multiplication.	**Solve & Share:** Watch for students that appear to be struggling with 4 x 50, as the basic fact ends with zero. For these students have them identify the basic fact product (e.g., 20). An instructional suggestion is offered in *Prevent Misconceptions* (TE, p. 552). After students have shared their solution methods and reasoning, consider discussing the *Look Back!* prompt. Ask students to generalize and create a rule for multiplying with 10s based on their observations of patterns in the products in the *Solve and Share* problems. **Visual Learning:** Consider discussing the *Convince Me!* prompt if you feel your students need an additional opportunity to apply the Associative Property of Multiplication for multiplying by multiples of 10. **Independent Practice/Math Practices and Problem Solving:** Consider assigning item 21 MP. 3 Critique Reasoning to give students the opportunity to reason about products made when multiplying by a multiple of 10, and how these ideas can be applied when the basic fact product ends in 0. Consider assigning item 23 *Algebra* to support students' reasoning with multiples of 10 when dividing. The strategy of thinking of division as an unknown factor problem can be very helpful in supporting students understanding of both operations. **Assess and Differentiate:** If time permits, you may consider replacing the *Math and Science Activity* with games from previous topics or the *Fluency Practice Activity* (TE, p. 563). Child-watch to identify students who need additional support and consider the *Intervention Activity* provided (TE, p. 555A). Consider utilizing the following question format during practice: **Example Stem 2:** What unknown number makes the equation true? 3 x □ = 180 Enter your answer in the response box. **Rubric:** (1 point) The student enters the correct product (e.g., 60). **Example Stem 4:** What unknown number makes the equation true? 60 x □ = 540 Enter your answer in the response box. **Rubric:** (1 point) The student enters the correct product (e.g., 9). **Example Stem 5:** What unknown number makes the equation true? 540 = □ x 60 Enter your answer in the response box. **Rubric:** (1 point) The student enters the correct product (e.g., 9).
Lesson 10-4: Math Practices and Problem Solving- Look for and Use Structure		
3.NBT.A.3 **MP.7** MP.1 MP.3 MP.4 MP.8	**Access Prior Learning:** In Grade 2, students learned how to use the structure of place value to compare numbers. **Securing the Big Idea:** In this lesson, students *secure* their understanding of strategies for multiplying by multiples of 10 by using the structure of the multiplication table and place value to solve problems.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 7, *"Look for and make use of structure."* Refer to the *Math Practices and Problem Solving Handbook* (TE, pp. F27-F27A, F29) for suggestions on how to develop, connect and assess this Math Practice. Also reference the handbook in the Student Edition (SE, p. F27). **Solve & Share:** Consider revisiting MP. 7 Thinking Habits (SE p. F27) before introducing the *Solve & Share*. Asking the questions in *Build Understanding* can help students to make sense of the problem as they work to solve. Consider using the time when students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.7 that are listed in the *Math Practices and Problem Solving Handbook* (TE, p. F27A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice. ***-continues on next page-***

		After students have had an opportunity to share, compare and discuss their solution methods and reasoning used for the *Solve and Share*, consider discussing the *Look Back!* prompt to extend opportunities for reasoning. Are students connecting ideas to place value and the properties of multiplication? Ask students to justify their explanations and if possible, generalize to create a rule for multiplying with multiples of 10. **Visual Learning:** Consider assigning the *Convince Me!* as it provides an opportunity for students to generalize the understanding discussed in the *Visual Learning Animation* and reason more with MP.7. **Assess and Differentiate:** If time permits, you may consider replacing the *Problem Solving Reading Mat* with games from previous topics or the *Fluency Practice Activity* (TE, p. 563). Child-watch to identify students who need additional support and consider the *Intervention Activity* provided (TE, p. 561A). ***CTC: *Quick Check*** (digital platform) Items 1 and 3

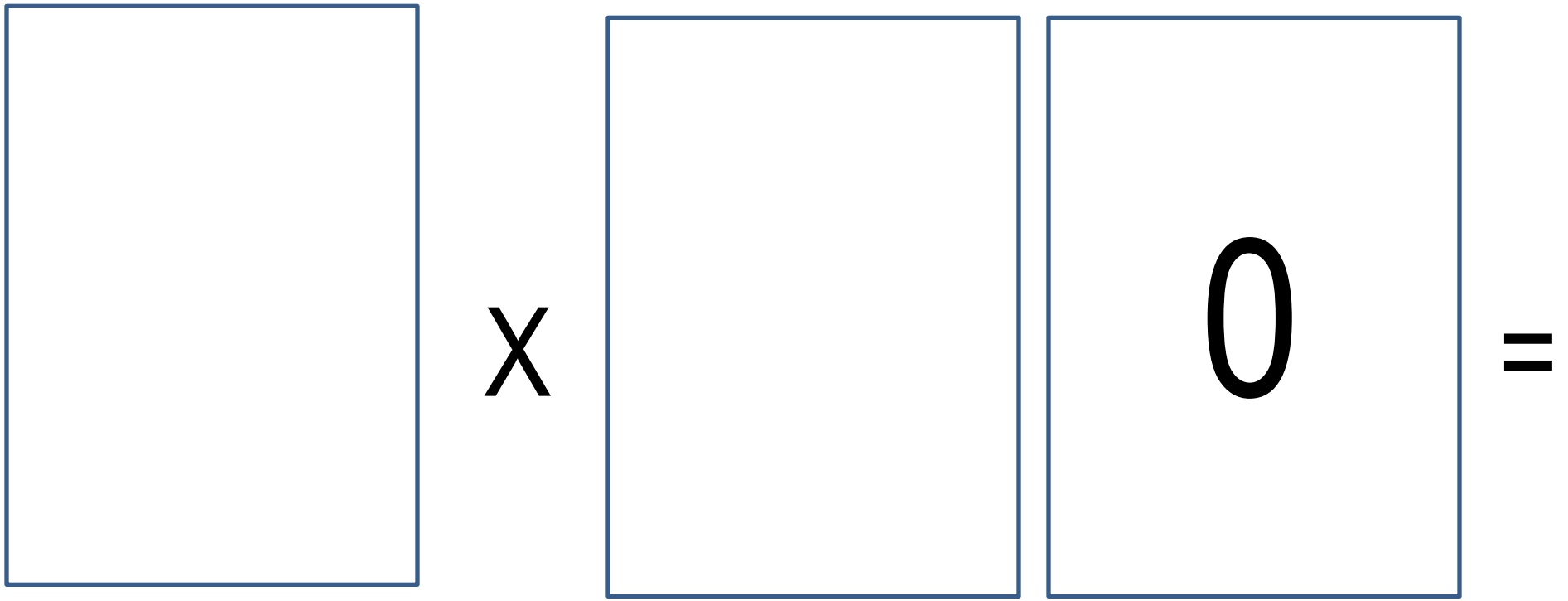
References

Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). K-5, Numbers in Operations Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf

K-5Math. Multiples of Ten Multiply. Retrieved June 6, 2018 from https://www.k-5mathteachingresources.com/support-files/multiples-of-ten-multiply.pdf

Multiply by Multiples of 10



Multiply by Multiples of 10
Sentence Frames

I know that _____ times _____ is _____, so _____ times tens equals _____ tens or _____.	I know that _____ times _____ is _____, so _____ times tens equals _____ tens or _____.
_____ times _____ means _____ groups of tens, which is _____ tens or _____.	_____ times _____ means _____ groups of tens, which is _____ tens or _____.

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